Supplemental Information for Effect and Pathways Modifiers in a Bayesian Pathways Analysis of the National Human Exposure Assessment Survey for Arsenic in EPA Region 5

Definitions of Input Variables and Fitted Bayesian Pathways Models

This document specifies the variables and results from all Bayesian Pathways Models fitted to the NHEXAS data in the paper *Effect and Pathways Modifiers in a Bayesian Pathways Analysis of the National Human Exposure Assessment Survey for Arsenic in EPA Region 5.* It also lists the full-conditional distributions of the model parameters used by our MCMC algorithm.

1 Media and Pathway Modifier Variables

See Tables 1 and 2.

2 Full Conditional Distributions of the Hierarchical Bayesian Subpopulation Model

2.1 Posterior distribution

Letting $\boldsymbol{\theta} = (\boldsymbol{\beta}, \boldsymbol{\tau}, \boldsymbol{\omega})$, the posterior distribution is

$$egin{array}{rcl} [heta,X|Y] &\propto & [Y|X, heta] \left[X| heta
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assuming independence of the three components within θ . The distribution is not available in closed form and so we use a Gibbs sampler Markov chain Monte Carlo (MCMC), based on an augmented (including censored and missing data) joint distribution to sample from this posterior distribution.

Variable	Type	Definition and Units
UR	М	natural log of arsenic concentration in Urine $(\mu g/\ell)$
FD	М	natural log of arsenic intake in Food $(\mu g/day)$
BV	М	natural log of arsenic intake in Beverage $(\mu g/day)$
IA	М	natural log of arsenic concentration in Indoor Air (ng/m^3)
OA	М	natural log of arsenic concentration in Outdoor Air (ng/m^3)
РА	М	natural log of arsenic concentration in Personal Air (ng/m^3)
SL	М	natural log of arsenic concentration in Soil (ng/g)
SD	М	natural log of arsenic loading in Surface (Sill) Dust $(\mu g/cm^2)$
WF	m	natural log of arsenic concentration in Tap Water, Flushed $(\mu g/\ell)$
logcrtst	С	natural log of creatinine concentration $(\mu g/\ell)$
m log crtn	С	negative of the natural log of creatinine concentration $(\mu g/\ell)$
logcreatinine	С	the natural log of creatinine concentration $(\mu g/\ell)$
male	C, E	binary $(0/1)$ variable indicating the participant is male
female	Е	binary $(0/1)$ variable indicating the participant is female
child4to11/Age1	C, E	binary $(0/1)$ variable indicating the participant is between 4 and 11 years old
age	C, E	participant's age (years)
AgeAtoB	C, E	binary $(0/1)$ variable indicating the participant's age is between A and B years old
Age2	C,E	binary $(0/1)$ variable indicating the participant's age is between 12 and 19 years
hhold	E, P	household size
log hhold	Е, Р	natural log of household size
minushhold	E, P	negative of household size
hrs.home	Е	average number of hours participant spends inside home/day
glass	Е	average number of glasses of water participant drinks/day
tap.c	Р	binary $(0/1)$ variable indicating participant cooks with tapwater
fruit	Р	binary $(0/1)$ variable indicating participant eats fruit at least 3 days/three-months
tap.d/tapwater	Е	binary $(0/1)$ variable indicating participant drinks tapwater
c.air	Е	binary $(0/1)$ variable indicating participant's home has central air
no.c.air	Е	binary $(0/1)$ variable indicating participant's home does <i>not</i> have central air
gasequip	Р	binary $(0/1)$ variable indicating participant used gas equipment in the past week

Table 1: For each variable used in the Bayesian hierarchical analysis, the variable symbol, its usedepending on the fitted model (M = media, C = component of the creatinine adjustment, E = effect modifier, and P = pathway modifier), and its definition, including units.

Variable	Type	Definition and Units
tobacco/tobacco1	Р	binary $(0/1)$ variable indicating the participant is a current smoker
tobacco2	Р	binary $(0/1)$ variable indicating the participant is a former smoker
workshop	Р	average number of minutes/day in an enclosed workshop
work20	Р	binary (0/1) variable indicating participant works ≥ 20 min./day in a workshop
fish	Р	binary $(0/1)$ variable indicating participant eats fish
new.fish	Р	binary $(0/1)$ variable indicating participant eats fish more than once/month
no.fish	Р	binary $(0/1)$ variable indicating participant eats fish less than once/month
logBMI	С	Natural log of Body Mass Index (kg/m^2)
race3_2	C, E	a binary $(0/1)$ variable indicating whether the participant is African-American
race3_3	C, E	binary $(0/1)$ variable indicating the participant's race is <i>not</i> Caucasian/African-American
no.wellwater	Р	binary $(0/1)$ variable indicating the source of running water is well water
wellwater	Р	binary $(0/1)$ variable indicating the source of running water is <i>not</i> well water

Table 2: For each variable used in the Bayesian hierarchical analysis, the variable symbol, its usedepending on the fitted model (M = media, C = component of the creatinine adjustment, E = effectmodifier, and P = pathway modifier), and its definition, including units.

2.2 Full conditional distributions

1. For each i and j we set/draw augmented data, $Y^{\ast}_{ij},$ as follows:

$$Y_{ij}^* \qquad \begin{cases} = Y_{ij}, & Z_{ij} = 0; \\ \sim \text{Truncated } N(X_{ij}, 1/\omega_j) \text{ on } (-\infty, M_{ij}], & Z_{ij} = 1; \\ \sim N(X_{ij}, 1/\omega_j), & Z_{ij} = 2. \end{cases}$$

2. For the process variable X, we sample each process variable X_j , j = 1..., J, conditional on the other process variables $\{X_k : k \neq j\}$ and the other parameters in the model. We have that

$$egin{aligned} & [oldsymbol{X}_j|\{oldsymbol{X}_k:k
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eq l\},oldsymbol{eta}_l, au_l], \end{aligned}$$

where ch_j denotes the columns of the process variable X that depend on X_j in their definition of the conditional mean μ_{ij} (i.e., those l for which $s_{lk}^X = j$ for some k or $s_{lm}^{EX} = j$ for some m). The above is expression is equal to

$$\prod_{i=1}^{I} \left(\left[Y_{ij}^* | X_{ij}, \omega_j \right] \left[X_{ij} | \{ X_{ik} : k \neq j \}, \boldsymbol{\beta}_j, \tau_j \right] \prod_{l \in ch_j} \left[X_{il} | \{ X_{ik} : k \neq l \}, \boldsymbol{\beta}_l, \tau_l \right] \right),$$

from which we can see that we can sample each X_{ij} , i = 1, ..., I, independently. Now

$$\begin{bmatrix} Y_{ij}^* | X_{ij}, \omega_j \end{bmatrix} \propto \exp\left(\frac{\omega_j}{2} (Y_{ij}^* - X_{ij})^2\right),$$
$$\begin{bmatrix} X_{ij} | \{X_{ik} : k \neq j\}, \beta_j, \tau_j \end{bmatrix} \propto \exp\left(\frac{\tau_j}{2} (X_{ij} - \mu_{ij})^2\right),$$

and for each $l \in ch_j$

$$[X_{il}|\{X_{ik}:k\neq l\},\boldsymbol{\beta}_l] \propto \exp\left(\frac{\tau_l}{2}(X_{il}-\delta_{il}(j)-\epsilon_{il}(j))^2\right).$$
(1)

In (1) we define

$$\delta_{ij}(o) = \alpha_j + \sum_{\{k:s_{jk}^X \neq o\}} \beta_{jk}^X X_{is_{jk}^X} + \sum_l \beta_{jl}^P P_{is_{jl}^P} + \sum_m \beta_{jm}^E E_{is_{jm}^E} + \sum_{\{n:s_{jn}^{EX} \neq o\}} \beta_{jn}^{EX} E_{is_{jn}^E} X_{is_{jn}^{EX}} + \sum_{ijn} \beta_{in}^{EX} E_{is_{jn}^E} X_{is_{jn}^E} + \sum_{ijn} \beta_{in}^{EX} E_{in} X_{is_{jn}^E} + \sum_{ijn} \beta_{in}^{EX} E_{in} X_{is_{jn}^E} + \sum_{ijn} \beta_{in}^{EX} E_{in} X_{in} X_{in} X_{in} + \sum_{ijn} \beta_{in}^{EX} E_{in} X_{in} X_{in} X_{in} + \sum_{ijn} \beta_{in}^{EX} E_{in} X_{in} X_{in}$$

to be the conditional mean of X_{ij} minus the regression terms that depend on X_{io} , and

$$\epsilon_{ij}(o) = \sum_{\{k:s_{jk}^X = o\}} \beta_{jk}^X X_{is_{jk}^X} + \sum_{\{m:s_{jm}^{EX} = o\}} \beta_{jm}^{EX} E_{is_{jm}^E} X_{is_{jm}^{EX}}$$
$$= X_{io} \left[\sum_{\{k:s_{jk}^X = o\}} \beta_{jk}^X + \sum_{\{m:s_{jm}^{EX} = o\}} \beta_{jm}^{EX} E_{is_{jm}^E} \right],$$

to be the regression terms that depend on X_{io} . Always, $\delta_{ij}(o) + \epsilon_{ij}(o) = \mu_{ij}$. Rearranging the terms of X_{ij} , the distribution of X_{ij} conditional on all other terms is N(q/p, 1/p) where

$$q = \omega_j Y_{ij}^* + \tau_j \mu_{ij} + \sum_{l \in ch_j} \tau_l \left(X_{il} - \delta_{il}(j) \right) \left(\sum_{\{k:s_{jk}^X = n\}} \beta_{jk}^X + \sum_{\{m:s_{jm}^{EX} = n\}} \beta_{jm}^{EX} E_{is_{jm}^E} \right)$$

and

$$p = \omega_j + \tau_j + \sum_{l \in ch_j} \tau_l \left(\sum_{\{k:s_{jk}^X = n\}} \beta_{jk}^X + \sum_{\{m:s_{jm}^{EX} = n\}} \beta_{jm}^{EX} E_{is_{jm}^E} \right)^2.$$

3. For the regression parameters β , we sample each β_j conditional on the other parameters in the model. Letting

$$\mathbf{A}_{j} = \begin{bmatrix} 1 & \left\{ X_{1s_{jk}^{X}} \right\} & \left\{ P_{1s_{jk}^{P}} \right\} & \left\{ E_{1s_{jk}^{E}} \right\} & \left\{ E_{1s_{jk}^{E}} X_{1s_{jk}^{EX}} \right\} \\ 1 & \left\{ X_{2s_{jk}^{X}} \right\} & \left\{ P_{2s_{jk}^{P}} \right\} & \left\{ E_{2s_{jk}^{E}} \right\} & \left\{ E_{2s_{jk}^{E}} X_{2s_{jk}^{EX}} \right\} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \left\{ X_{Is_{jk}^{X}} \right\} & \left\{ P_{Is_{jk}^{P}} \right\} & \left\{ E_{Is_{jk}^{E}} \right\} & \left\{ E_{Is_{jk}^{E}} X_{Is_{jk}^{EX}} \right\} \end{bmatrix},$$

the distribution of $\boldsymbol{\beta}_j$ conditional on the other parameters is

$$\operatorname{MVN}_{r_j}\left(\boldsymbol{R}_j^{-1}\boldsymbol{q}_j, \boldsymbol{R}_j^{-1}\right),$$

where $\boldsymbol{R}_j = \tau_j \boldsymbol{A}_j^T \boldsymbol{A}_j + \boldsymbol{\Sigma}_j^{-1}$ and $\boldsymbol{q}_j = \tau_j \boldsymbol{A}_j^T \boldsymbol{X}_j + \boldsymbol{\Sigma}_j^{-1} \boldsymbol{m}_j$.

4. For each j, the conditional distribution of the process precision τ_j , given the other parameters, is

$$Ga\left(c_j + I/2, \ d_j + \sum_{i=1}^N (X_{ij} - \mu_{ij})^2/2\right),$$

where μ_{ij} was defined above in Equation (1) of the paper.

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Models fit in "Effect and Pathways Modifiers in a Bayesian Pathways Analysis of the NHEXAS for Arsenic in EPA Region 5"

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Models fit in "Effect and Pathways Modifiers in a Bayesian Pathways Analysis of the NHEXAS for Arsenic in EPA Region 5"

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