1. [10 points] Please refer to the Problem 1 on the Take-Home Portion.
   a. [5 points] If the column rank of the design matrix $X$ is not full, find the estimable linear function(s) of $\beta_1$ and $\beta_2$.
   b. [5 points] Let $\hat{\beta}_1$ denote the ordinary least squares (OLS) estimator of $\beta_1$. Obtain the efficiency of $\hat{\beta}_1$ relative to $\hat{\beta}_1$, i.e., $\text{var}(\hat{\beta}_1) / \text{var}(\hat{\beta}_1)$.

2. [15 points] Please refer to the Problem 2 on the Take-Home Portion.
   a. [5 points] Show that the Pure Error Sum of Squares, based on $\{(Y_{ijk} - \bar{Y}_j)_i, i, j, k = 1, 2\}$, and the BLUE of $\tau_1 - \tau_2$ are independently distributed.
   b. [5 points] Find the variances of the OLS and BLUE estimators of $\tau_1 - \tau_2$ in part e.
   c. [5 points] Explain the differences, if any, between the F-tests corresponding to the full model and the model without interactions in part f.

3. [5 points] Please refer to the Problem 4 on the Take-Home Portion. Show that each component of $K'b$ in part e of the problem is estimable (so that the hypothesis is testable).

4. [10 points] Explain why each of the following statements is valid or not!
   a. [2 points] Let $\hat{Y}$ denote the projection of $Y$ onto the column space of the matrix $X$ in a general linear model. The correlation between $\hat{Y}$ and $e = Y - \hat{Y}$ is always positive.
   b. [2 points] The $F$-test of a non-testable hypothesis $H_0 : K'\beta = m$, just ignores the linear functions in $H_0$ that do not belong to the space $C(X')$.
   c. [2 points] If the $F$-test rejects the hypotheses $H_0 : \tau_1 = \tau_2 = \cdots = \tau_k$ in an ANOVA model, then at least one pair-wise difference must be declared as non-zero according to the Bonferroni Method.
   d. [2 points] Refer to Problem 5, part b on the Take-Home Portion. Let $A$ denote the matrix in the quadratic form $Q = YAY$. Then the matrix $A$ is non-negative definite.
   e. [2 points] Refer to Problem 3, part b on the Take-Home Portion. For any vector $l$ such that $l \perp C(X)$, the conditional distribution of $l'Y$ given $XY$ is the same as its marginal distribution.