Geostatistical prediction of spatial extremes and their extent

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Abstract

Suppose we are interested in making inference on the spatial process $\{Z(\mathbf{s})\colon \mathbf{s} \in D \subset \mathbb{R}^d\}$ based on a sample $\mathbf{Z} \equiv (Z(\mathbf{s}_1), \ldots, Z(\mathbf{s}_n))'$ taken at locations $\{\mathbf{s}_1, , \mathbf{s}_n\}$. It is well known that optimal linear prediction of linear functionals of Z (e.g., $Z(\mathbf{s}_0)$ at a known point $\mathbf{s}_0$ or $Z(B) \equiv \int_B Z(\mathbf{u})d\mathbf{u}/|B|$ over a given block B in D) is obtained through (universal) kriging. When the functional of Z is nonlinear, such as the α-th spatial quantile in D, $Z_\alpha(D) \int \inf\{z\colon \int_D I(Z(\mathbf{u}) \leq z)d\mathbf{u}/|D| \geq \alpha\}$; α near 1, a plug-in kriging predictor is highly biased. The reason is that kriging is based on a squared error loss function that results in "oversmoothing"; that is, the peaks of Z are rounded and the valleys of Z are filled in. One way to deal with this is to add a constraint that matches certain variances and covariances of the predictor to that of Z: covariance-matching constrained kriging (CMCK). The other way is to change the loss function to put more emphasis on the peaks (and valleys): integrated weighted quantile squared error loss (IWQSEL) prediction. Craigmile et al. (2004) have shown that IWQSEL prediction can target a given value of α and accurately predict extremes and their extent associated with that α. In this paper, we compare CMCK to the IWQSEL-prediction standard for a given α, using a dataset of TCDD-contaminated soil.