Homework 5

Problem 1 Consider the process $\{\varepsilon(\boldsymbol{s}) : \boldsymbol{s} \in D\}$ such that $E(\varepsilon(\boldsymbol{s})) = 0$, $\text{var}(\varepsilon(\boldsymbol{s})) = \sigma^2$, and if $\boldsymbol{s}_1 \neq \boldsymbol{s}_2$ then $\varepsilon(\boldsymbol{s}_1)$ is uncorrelated with $\varepsilon(\boldsymbol{s}_2)$. Find the variogram $2\gamma(\boldsymbol{h})$ of the process $\varepsilon(\cdot)$.

Problem 2 Consider the process $\{\varepsilon(s) : s \geq 0\}$, defined in one dimension, as in question 1. Let

$$W(s) = \int_0^s \varepsilon(u)du\,; \qquad s \geq 0\,.$$

(a) Show that $E(W(s)) = 0$ and $\text{cov}(W(s), W(t)) = \sigma^2 \min(s,t)$.

(b) Find $\text{var}(W(s) - W(t))$.

(c) Comment on whether the process $W(\cdot)$ is second-order stationary or intrinsically stationary.

Problem 3 Consider a spatial random process $\{Z(\boldsymbol{s}) : \boldsymbol{s} \in D\}$ and assume that its mean and covariance function are *known*. That is, asume that μ and $C(\cdot,\cdot)$ are

$$\mu \equiv E(Z(\boldsymbol{s}))\,; \qquad \boldsymbol{s} \in D$$

$$C(\boldsymbol{s},\boldsymbol{u}) \equiv \text{cov}(Z(\boldsymbol{s}), Z(\boldsymbol{u}))\,; \qquad \boldsymbol{s},\boldsymbol{u} \in D\,.$$

Suppose we want to predict $Z(\boldsymbol{s}_0)$ from data $\boldsymbol{Z} \equiv (Z(\boldsymbol{s}_1), \ldots, Z(\boldsymbol{s}_n))'$ using the linear predictor

$$Z^*(\boldsymbol{s}_0) = \sum_{i=1}^{n} \ell_i Z(\boldsymbol{s}_i) + c\,.$$

(a) Find $\ell_1, \ldots, \ell_n$, c to minimize

$$\text{MSPE} \equiv E(Z(\boldsymbol{s}_0) - Z^*(\boldsymbol{s}_0))^2\,.$$

(b) Notice that there are no restrictions in (i) to guarantee that $Z^*(\boldsymbol{s}_0)$ is unbiased. Show that the solution to (i) is indeed unbiased; that is, show

$$E(Z^*(\boldsymbol{s}_0)) = E(Z(\boldsymbol{s}_0))\,.$$