Stat 693 C **Final Exam (90 minutes)** **Spring, 2000**

1. (7 points) In half a page, say what makes an environmental study different from an ecological study, and give an example of each.

2. (6 points) In 1965, in *Proc. Roy. Soc. of Medicine*, Bradford Hill discussed the circumstances under which an effect obtained in an observational study is relatively likely to have a causal interpretation. Give three such circumstances that the effect should have, namely, the effect should be a) _______________, b) _______________, and c) _______________.

3. (8 points) Suppose that the observation Z and the true value $S > 0$ are linked through *multiplicative* measurement error:

$$Z \equiv S \cdot (\sigma \varepsilon)\,,$$

where random variables S and ε are independent and ε has probability density function (pdf) $f^{(\varepsilon)}$. Derive the conditional pdf $f^{(Z|S)}(z|s)$ in terms of $f^{(\varepsilon)}$ and σ^2.

4. (4 points) Bayes' Theorem allows one to calculate $f^{(S|Z)}(s|z)$ from $f^{(Z|S)}(z|s)$ and $f^{(S)}(s)$. Give the formula.

5. (a) (4 points) Suppose that the random variable S is lognormally distributed with parameters μ and τ^2. Show that the pdf of S is:

$$f^{(S)}(s) = s^{-1}(2\pi\tau^2)^{-1/2}\exp\{-(\log s - \mu)^2/(2\tau^2)\}\,.$$

   (b) (5 points) Use Questions 3, 4, and 5a) to give a formula for $f^{(S|Z)}(s \mid z)$ in terms of $f^{(\varepsilon)}$, μ, σ^2, and τ^2.

   (c) (2 points) Give an example of an environmental study where you would use $f^{(S|Z)}$.

6. (a) (12 points) Consider the intrinsically stationary process $Z(\cdot)$ with variogram $2\gamma(\mathbf{h})$. Suppose that $Z(\cdot)$ satisfies the additive-measurement-error model:

$$Z(\mathbf{s}) = S(\mathbf{s}) + \sigma\varepsilon(\mathbf{s})\,; \qquad \mathbf{s} \in D\,,$$

where $\varepsilon(\cdot)$ is a zero-mean, white-noise process. Suppose we wish to predict $S(\mathbf{s}_0)$ based on data $Z(\mathbf{s}_1), \ldots, Z(\mathbf{s}_n)$. We use the ordinary-kriging predictor,

$$\widehat{S}(\mathbf{s}_0) = \sum_{i=1}^{n} \nu_i Z(\mathbf{s}_i)\,,$$

where $E(\widehat{S}(\mathbf{s}_0)) = E(S(\mathbf{s}_0))$ implies $\sum_{i=1}^{n} \nu_i = 1$. Show that the ordinary-kriging prediction equation for $\boldsymbol{\nu}$ is given by,

$$\begin{aligned} \Gamma\boldsymbol{\nu} + \mathbf{1}m &= \boldsymbol{\gamma}^* \\ \mathbf{1}'\boldsymbol{\nu} &= 1\,, \end{aligned}$$

where Γ is an $n \times n$ matrix with (i,j)th element $\gamma(\mathbf{s}_i - \mathbf{s}_j)$, $\mathbf{1} = (1, \ldots, 1)'$, $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_n)'$, m is a Lagrange multiplier, $\boldsymbol{\gamma}^* = (\gamma^*(\mathbf{s}_0 - \mathbf{s}_1), \ldots, \gamma^*(\mathbf{s}_0 - \mathbf{s}_n))'$, and

$$\gamma^*(\mathbf{h}) = \begin{cases} \gamma(\mathbf{h})\,; & \mathbf{h} \neq \mathbf{0} \\ \\ \sigma^2\,; & \mathbf{h} = \mathbf{0}\,. \end{cases}$$

(b) (2 points) What is the difference between the kriging equation for $\boldsymbol{\lambda}$ in $\widehat{Z}(\mathbf{s}_0) = \sum_{i=1}^{n} \lambda_i Z(\mathbf{s}_i)$ given in lectures, and that for $\boldsymbol{\nu}$ in $\widehat{S}(\mathbf{s}_0)$ given in part a)?

(c) (2 points) What effect does this difference have on predicting $Z(\mathbf{s}_0)$ or $S(\mathbf{s}_0)$ when $\mathbf{s}_0 = \mathbf{s}_1$?