

1. Please use a paragraph to respond to each of the following questions. Use knowledge gained from class plus any other sources you might have.
 - a. What is the role of environmental statistics in studying the environment?
 - b. What is the role of satellite data in environmental science?
 - c. What is the role of spatial statistics in environmental statistical problems?

2. Suppose that data $\{Y_i: i = 1, \dots, n\}$ are observed along with concomitant information $\{x_i: i = 1, \dots, n\}$. In order to capture the variability in Y , a linear regression of Y on x is modeled:

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i; \quad i = 1, \dots, n,$$

where the errors $\{\varepsilon_i\}$ are assumed independent and identically distributed (i.i.d.) $N(0, \sigma^2)$; σ^2 unknown.

- a. Derive formulas for the ordinary least squares estimates $\hat{\beta}_0$ and $\hat{\beta}_1$.
 - b. Define the residual sum of squares and derive an unbiased estimator of σ^2 based on it; call it $\hat{\sigma}^2$. Prove that $\hat{\sigma}^2$ is unbiased.
 - c. Give a size α test of the hypotheses $H_0: \beta_1 = 0$ versus $H_1: \beta_1 \neq 0$. [Hint: The test statistics should be based on estimates in a. and b.]
 - d. Give a $100(1 - \alpha)\%$ confidence interval for β_1 .
3. Suppose Y is Poisson with mean λ . Show that

$$E[1/(Y + 1)] = \{1 - \exp(-\lambda)\}/\lambda.$$

4. Let Y_1, \dots, Y_n be i.i.d. with density

$$f_\phi(y) = \phi^{-1} \exp\{-y/\phi\} I(y \geq 0),$$

where $\phi > 0$ and $I(A)$ is the indicator function equal to 1 if A is true and equal to 0 otherwise. Give the maximum likelihood estimate for ϕ .