Asymptotic Properties of Covariance-Matching Constrained Kriging

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Consider a random field $S(\mathbf{s})$, where $\mathbf{s}$ belongs to a domain D in d-dimensional Euclidean space,

$$S(\mathbf{s}) = \mu(\mathbf{s}) + d(\mathbf{s}).$$

Here $\mu(\cdot)$ is the large-scale, deterministic, mean structure of the signal $S(\cdot)$, and $d(\cdot)$ is the small-scale stochastic structure that models te spatial dependence. The $S(\cdot)$ represents the spatial process of fundamental interest, namely the signal. The observations of $S(\cdot)$ at discrete points are contaminated with additive measurement error, according to the model

$$Z(\mathbf{s}) = S(\mathbf{s}) + \epsilon(\mathbf{s}).$$

Here $\epsilon(\mathbf{s})$ is a zero-mean, white-noise process representing measurement error. Following Aldworth and Cressie (2003) we investigate a predictor of nonlinear functional $g(S(\cdot))$. The predictor is based on the kriging methodology with extra constraints and called *covariance-matching constrained kriging.* It is an optimal linear predictor that matches not only first moments but second moments as well.

The underlying optimization problem is linear programming problem, with both linear and quadratic constraints. By geometric reasoning we show that the problem always has a solution provided the constraints are consistent, and propose a new algorithm to compute a solution.

We prove that the predictor is asymptotically unbiased. We give an upper bound for the L1-error of prediction. In general the error is not vanishing as the sample size tends to infinity, but it is asymptotically small if the second and third derivatives of nonlinear tranform $g(\cdot)$ are small enough.

References
Aldworth, J., and Cressie, N., 2003. Prediction of nonlinear spatial functions. J. Statist. Plann. and Infer., 112, 3-41.