# Stat 882: Statistical Phylogenetics - Lecture 3 

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## 1 Introduction: Optimality Criteria

## Optimality Criteria

- Thus far, we have described two criteria for evaluating phylogenetic trees
- Parsimony: Prefer the tree(s) that minimize evolutionary change
* Compute a score for each tree based on a cost associated with each type of change
- Maximum likelihood: Prefer the tree(s) that have the highest likelihood
* Compute the likelihood of each tree under a particular substitution model
- Using these criteria, we have a method for comparing different phylogenies

Optimality Criteria - Inference

- Recall that our goal is to infer a phylogeny
- To do this, we must search the set of possible phylogenies for the tree(s) that gives the best value of the selected criterion
- Recall that the space of phylogenies is very large - consider an unrooted tree with $T$ taxa. There are
- $T-2$ internal nodes
$-2 T-3$ branches
- Number of possible trees is $\prod_{i=1}^{T-2}(2 i-1)$

| Number of <br> Taxa | Number of <br> Rooted Trees | Number of <br> Unrooted Trees |
| :---: | :---: | :---: |
| 5 | 105 | 15 |
| 10 | $34,459,425$ | $2,027,025$ |
| 20 | $8.2 \times 10^{21}$ | $2.21 \times 10^{20}$ |
| 50 | $2.75 \times 10^{76}$ | $2.83 \times 10^{74}$ |

## 2 Methods for Finding Optimal Trees: Exact Methods

Methods for Finding Optimal Trees

- Exact Methods
- Exhaustive search
- Branch and bound methods
- Heuristic methods
- Divide-and-conquer
- Stepwise addition and branch swapping
- Parsimony ratchet
- Numerous other possibilities
- Stochastic searches
- Simulated annealing
- Genetic algorithms


### 2.1 Exhaustive Search

## Exhaustive Search

- Enumerate all possible trees
- Evaluate the criterion of interest on all trees
- Pick the tree that gives the optimal value of the criterion
- Advantage: Complete information about the problem
- Disadvantage: Not computationally feasible for large numbers of taxa


### 2.2 Branch and Bound

## Branch and Bound

- Can be used with any criterion whose values are non-decreasing as taxa are added to the tree
- Basic idea: Eliminate portions of the tree space that do not contain the optimal tree, so that the criterion need never be evaluated for these trees
- Advantage: Guaranteed to find optimal tree
- Disadvantages:
- Don't necessarily give info about near-optimal trees
- May still be very time consuming
- Limited to approx. 20 taxa or less


## Branch and Bound - An Example

- Consider the following data simple set:

| Taxon | Site Pattern |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Name | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |
| 1 | A | A | C | C | A | C | A |  |
| 2 | A | T | C | G | T | G | C |  |
| 3 | A | T | C | G | A | C | A |  |
| 4 | C | T | T | T | T | C | C |  |
| 5 | C | G | T | T | T | C | C |  |

- Suppose we want to find the tree with the lowest score, $S$, under Fitch parsimony


## Branch and Bound - An Example

- Use an initial, quick search to find a tree whose length is 9
- Begin with three taxa, then add the fourth in all possible locations; evaluate all scores



## Branch and Bound - An Example

- Then add the fifth taxon



## Branch and Bound - Worst Case

- Suppose instead our data were only the first 6 sites:

| Taxon | Site Pattern |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Name | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | A | A | C | C | A | C |
| 2 | A | T | C | G | T | G |
| 3 | A | T | C | G | A | C |
| 4 | C | T | T | T | T | C |
| 5 | C | G | T | T | T | C |

## Branch and Bound - An Example

- Use an initial, quick search to find a tree whose length is 8
- Begin with three taxa, then add the fourth in all possible locations; evaluate all scores


Branch and Bound - An Example

- Then add the fifth taxon - note that now all trees must be evaluated



## 3 Methods for Finding Optimal Trees: Heuristic Methods

### 3.1 Divide-and-conquer Methods

Divide-and-Conquer Methods

- Divide the collection of taxa into subgroups
- Infer optimal phylogenetic trees for each of the subgroups
- Reassemble the subtrees into an overall supertree that includes all of the taxa
- Advantage: Quick
- Disadvantages:
- How do we divide the taxa into subgroups?
- How do we create supertrees from subtrees, especially if there is incongruence?


## Divide-and-Conquer Methods

- Some of the most widely-used methods in this class are:
- Disk-covering methods - Tandy Warnow's group Reference: U. Roshan, B. M. E. Moret, T. L. Williams, T. Warnow, Rec-I-DCM3: A Fast Algorithmic Technique for Reconstructing Large Phylogenetic Trees, Proceedings of the IEEE Computational Systems Bioinformatics (CSB04) Stanford (CA), USA, 2004
- Quartet puzzling - implemented in PAUP* Reference: Strimmer, K. and A. von Haeseler. 1996. Quartet Puzzling: A Quartet MaximumLikelihood Method for Reconstructing Tree Topologies. Molecular Biology and Evolution, Vol 13(7): 964-969.


### 3.2 Stepwise Addition and Branch Swapping

## Stepwise Addition and Branch Swapping

- Most commonly used estimation method (PHYLIP, PAUP*, fastDNAML)
- Form an initial tree by stepwise addition of taxa
- At each step, perform rearrangements of the tree until no improvement in optimality criterion can be made
- Several strategies for performing rearrangements
- Nearest Neighbor Interchagnes (NNI)
- Subtree pruning and regrafting (SPR)
- Tree bisection and reconnectin (TBR)


## Stepwise Addition and Branch Swapping

- Advantages:
- Easy to understand
- Implemented in software
- Works well for small sample sizes - e.g., $<50$ taxon
- Disadvantages:
- No guarantee of finding optimal tree (order of addition of taxa matters)
- One proposed solution: Perform the search from several starting points (orderings of the taxa)
- Computationally intensive


### 3.3 Ratchet

## Ratchet

- Originally proposed for parsimony (Kevin Nixon, 1999, Cladistics)
- Extended to likelihood (Rutger Vos, 2003, Systematic Biology)
- Alogrithm:
- Generate a starting tree by some quick method
- Reweight a randomly selected subset of characters (e.g., give a weight of 2 to $50 \%$ of the characters and a weight of 1 to the other $50 \%$ )
- Search on the current tree using any strategy (e.g., branch swapping)
- Set all characters back to their original weights and search again for the current tree
- Repeat these steps for many iterations


## Ratchet

- Advantages:
- Reweighting of characters allows the algorithm to explore more of the tree space than standard heuristics
- Gain information about locally optimal trees
- Easily implemented in PAUP* - Sikes and Lewis, PAUPRat
- Disadvantages:
- Need to decide how many iterations to use and how weights should be selected
- No guarantee of finding optimal tree


## 4 Methods for Finding Optimal Trees: Stochastic Methods

### 4.1 Simulated Annealing

## Simulated Annealing

- A general method of function optimization
- Basic idea:
- Move through the space of all trees by randomly rearranging a current tree to form a new tree.
- The criterion of interest is evaluated on the new tree, and a decision is made about whether the new tree should be accepted as the current tree.
- The key is that even new trees with worse values of the optimality criterion can be accepted - hopefully, this helps avoid finding only local optima.


## Simulated Annealing

- Three steps

1. From tree $\tau_{i}$, generate candidate tree $\tau^{*}$ via a randomly-selected NNI move.
2. If $L\left(\tau^{*}\right) \geq L\left(\tau_{i}\right)$, set $\tau_{i}=\tau^{*}$. Otherwise, set $\tau_{i}=\tau^{*}$ with probability $\exp \left\{\frac{L\left(\tau^{*}\right)-L\left(\tau_{i}\right)}{c_{i}}\right\}$.
3. Update the value of the control parameter, $c_{i}$, and set $i$ to $i+1$. Go to step 1.

## Simulated Annealing

- Advantages:
- Quick, and can handle large data sets
- Increased ability to find globally optimal tree
- Easy to implement
- Gives information about many trees
- Disadvantages:
- Many parameters must be specified
- No guarantee of finding optimal trees


### 4.2 Genetic Algorithms

## Genetic Algorithms

- A general method of function optimization
- Basic idea:
- Model the search for the optimal phylogenetic tree after the process of natural selection. Natural selection allows differential survival rates of individuals based upon their fitness.
- This is applied to the tree search problem by letting a particular trees fitness be represented by its value of the optimality criterion. Trees with better values of the criteria are more likely to proceed to the next generation.
- After many generations are simulated, we will hopefully have found the optimal tree.


## Genetic Algorithms

- Some details:
- Begin with a population of trees
- Compute values of the optimality criterion for each tree
- Mutation, natural selection, recombination, etc., act on each generation of the population to produce the next generation
* Examples: Branch lengths may be mutated by multiplication by a factor selected from a gamma distribution Topology might be mutated by performing an SPR rearrangement
- The tree with the highest value of the criterion is automatically placed in the next generation
- Repeat the process for many generations


## Genetic Algorithms

- Advantages:
- Quick
- Increased ability to find globally optimal trees
- Potential information about many other trees
- Implemented in the user-friendly program GARLI (Zwickl, 2006)
- Disadvantages:
- Many parameters must be specified
- No guarantee of finding optimal trees

A few other programs to mention ....

- RAxML - Randomly Axelerated Maximum Likelihood
- Author: Alexandros Stamatakis
- Website: http://icwww.epfl.ch/~stamatak/index-Dateien/Page443.htm
- TNT - Tree analysis using New Technology
- Author: Pablo Goloboff, James Farris, Kevin Nixon
- Website: http://www.cladistics.org/tnt.html

