Key Concepts in Linear Algebra

I. Notation s: Vectors and Vector Spaces

- *n*-dimensional vector: Ordered *n*-tuple
- Notation: $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, where $a_i \in \Re$ (real-valued elements) is a 3-dim column vector. A row vector is denoted by $\mathbf{a}' = (a_1 \quad a_2 \quad a_3)$
- Vector Space: $V_n = \{\mathbf{v}_i = (v_{i1} \quad v_{i2} \quad \cdots \quad v_{in})', v_{ij} \in \Re, i = 1, 2, \cdots\}$,
- $\Re^n$: *n*-dimensional Euclidean space
- i[th] elementary vector : $\mathbf{e}_i = (0, 0, \cdots, 1, 0, \cdots 0)'$
- $\mathbf{1}_n = (1 \quad 1 \quad \cdots \quad 1 \quad 1)'$

II. Linear independence, linearly independent vectors, basis of a vector space, subspaces, dimension (rank) of a vector space, space spanned by a collection of vectors

III. Inner (dot) Product of two vectors $\mathbf{x}, \mathbf{y} \in V_n$:

$$\mathbf{x} \bullet \mathbf{y} = \mathbf{x}'\mathbf{y} = \sum_i x_i y_i = \mathbf{y}'\mathbf{x} = \mathbf{y} \bullet \mathbf{x}$$

- Other notations: $(\mathbf{x}, \mathbf{y}), \langle \mathbf{x}, \mathbf{y} \rangle$
  - Kernel function:
    $k(\mathbf{x}, \mathbf{y}) = \langle \phi(\mathbf{x}), \phi(\mathbf{y}) \rangle$, where $\phi: \mathbf{x} \mapsto \phi(\mathbf{x}) \in F$, the feature space.
- Length of a vector, vector norm
- Angle between two vectors,
- Mutually orthogonal vectors,
- Pythagoras' theorem, Triangle inequality, Cauchy-Schwartz inequality,
- Orthogonal basis, orthonormal basis.
- Coordinates of a vector in terms of an o.n.b.
- Gram-Schmidt process for forming an o.n.b., Extension of an o.n.b. for a subspace *V* to an o.n.b. for the space V_n.
- Orthogonal Complement of a vector space

IV. Matrices (definition, sums and products, transpose,
   - Rank of a matrix
   - Space spanned by columns of a matrix **A**,
   - Null space of a matrix,
   - Matrix norm

V. Trace, Determinant, Eigen–values and Eigen-vectors and their properties, Spectral decomposition of a real symmetric matrix A.

VI. Singular Value Decomposition of a matrix A (SVD).

VII. Simultaneous diagonalization of two matrices.

VIII. Symmetric and Idempotent matrices, Partitioned matrices.

IX. Linear forms, Quadratic forms, Positive semi-definite (p.s.d) (n.n.d.), Positive Definite (p.d.).

X. Systems of Linear Equations (Solution space), generalized inverse of a matrix, computation of a g-inverse, Moore-Penrose inverse of a matrix.