STAT 742 Home Work Assignment #4 Goel

General Linear Model - Estimation

Problems to be submitted for the Assignment:

1) Text book, Chapter 3 Exercises #4.1, #4.2
2) Text book, Chapter 3 Exercises #4.4
3) Text book, Chapter 3 Exercise #4.17
4) Text book, Chapter 3 Exercise #4.19
5) Text book, Chapter 3 Exercise #4.26
6) Text book, Chapter 3 Exercise #4.28
7) Consider the general linear model $\mathbf{Y} = \mathbf{X}\beta + \varepsilon, E[\varepsilon] = 0, Var[\varepsilon] = \sigma^2 V,$ where ***V*** is a known positive definite matrix. Let ***M*** be an arbitrary, symmetric positive definite matrix, and let $\tilde{\beta}_{\mathrm{M}}$ denote a solution to the following optimization problem:

$$\min_{\beta}(\mathbf{Y} - \mathbf{X}\beta)'\mathbf{M}(\mathbf{Y} - \mathbf{X}\beta).$$

Note that this solution is not unique when ***X*** is not of full column rank. Note that the generalized least square estimator $\tilde{\beta}_{GLS}$ corresponds to $\mathbf{M} = \mathbf{V}^{-1}$.
Let $l'\beta$ be any estimable function.

- Show that $E[l'\tilde{\beta}_M] = E[l'\tilde{\beta}_{GLS}]$
- Show that $Var(l'\tilde{\beta}_M) \geq Var[l'\tilde{\beta}_{GLS}]$.

[Remarks: Please do not assume that $X'MX$ is invertible. Note that $l'\tilde{\beta}_I$ is the ordinary least squares (OLS) estimator and $l'\tilde{\beta}_{GLS}$ is the Gauss-Markov estimator of an estimable function $l'\beta$.]

Try to work on as many of the following Unassigned Problems as possible:

1) Text book, Chapter 4 Exercises #4.6, #4.7, #4.9, #4.11, #4.12, #4.13, #4.17, #4.20, #4.21