STAT 742 Home Work Assignment #1 Goel

Review Material on Vector Spaces and Matrices

Problems to be submitted for the Assignment:

1) Text book, Appendix A, Exercises A.1, A.12, A.16
2) Text book, Appendix A, Exercises A18, A19. Check if your basis vectors in the two problems mutually orthogonal. Find an orthonormal basis for each of these spaces.
3) Text book, Appendix A, Exercises A32
4) Let $\mathbf{X}_{n\times p}$ be an arbitrary matrix. Show that $\mathbf{PX'X} = \mathbf{QX'X} \Rightarrow \mathbf{PX'} = \mathbf{QX'}$.
5) Text book, Appendix A, Exercises A.36, A.44, A.49, A.50
6) Text book, Appendix A, Exercises A.53, A.54, A.73
7) Let $\mathbf{A}_{n\times n}$ be a non-singular matrix and $\mathbf{C}_{n\times m}$ be an arbitrary matrix. Let $\mathbf{B}^-$ denote a generalized inverse of a matrix $\mathbf{B}$. Prove that a generalized inverse of $(\mathbf{A}+\mathbf{CC'})$ is given by $(\mathbf{A}+\mathbf{CC'})^- = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{C}(\mathbf{I}+\mathbf{C'A}^{-1}\mathbf{C})^-\mathbf{C'A}^{-1}$.

Try to work on as many of the following Unassigned Problems as possible:

- Text book, Chap. 1 Exercises, A.2, A.5, A.6, A.7, A.22, A.23, A.24, A.34, A.4, A.42, A.43, A.52, A.60, A.72,
- Let $\mathbf{A}_{n\times n}$ be a symmetric matrix of rank $(n\text{-}1)$ such that $\mathbf{A}1 = 0.$ Show that $\mathbf{B} = \mathbf{A} + \frac{1}{n}\mathbf{11'}$ is non-singular and its inverse is given by $\mathbf{A}^+ + \frac{1}{n}\mathbf{11'},$ where $\mathbf{A}^+$ denotes the Moore-Penrose generalized inverse of $\mathbf{A}$.