## Statistics 742 Theory of Linear and Mixed Models — Autumn 2008

**Course Web Site:** http://www.stat.osu.edu/~goel

**Instructor:** Prof. Prem Goel
204C Cockins Hall, 292-8110
**E-mail:** goel@stat.osu.edu
**Office Hours**: M, W 11:30-12:30

**Graduate Teaching Assistant: Aaron Quan**
**Math Building, Room 417**
E-mail: quan.19@osu.edu
Office Hours: M, W 3:30-4:30.
Alternate times: Tues, Thur 10:00-11:0 (If no one from his other class is there.)

**Course Objectives:**

- Develop a critical understanding of the theoretical basis of statistical methods for linear models.
    - *Build a foundation for all statistical modeling, including Dynamic linear models, Generalized linear models, Econometric Models, Non-linear models and Time Series Analysis*
    - *Extensive problem solving is critical for a thorough understanding of a wide range of topics.*
    - *Your active involvement in-class, in solving Home Work problems, as well as additional problems in Supplementary Texts will help you achieve this goal.*

**Prerequisites:**

- Knowledge of basic theoretical statistics at the level of Stat 620-622
- Linear Algebra (Finite dimensional vector spaces and matrices): Stat 656 or Math 568

**Desirable Background:**

- Prior exposure to applied regression analysis and design of experiments (e.g., Stat 645 and 641) is highly desirable.

**Text Book:**

- Monahan, John F. (2008) *A Primer on Linear Models*, Chapman and Hall/CRC.

**Home Work Assignments:**

- Assigned problems will include further details on important theoretical facts and examples illustrating the theory outlined in lecture.
- You are encouraged to discuss homework problems in study teams. However, you must write your solutions independently of others.
- Solutions to Home work and the Midterm Exam will be posted on the website.

- **Home work assignments schedule:**

| HW # | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Assignment date** | **Thur 10/02** | **Thur 10/09** | **Thur 10/16** | **Thur 10/23** | **Thur 11/06** | **Thur 11/13** | **Thur 11/20** |
| **Due date** | **Thur 10/09** | **Thur 10/16** | **Thur 10/23** | **Thur 11/06** | **Thur 11/13** | **Thur 11/20** | **Thur 12/04** |

- **The lowest score will not be counted towards the grade.**
- **Please submit your home work upon entering the classroom**.
- **To be fair to everyone, late home work submission will not be accepted.**

**Examinations: The course will have two Examinations as follows:**

- **Midterm Exam: In Class on Thursday, October 30** (Closed book, Closed notes. One 8.5”x11” two-sided cheat sheet allowed.)
- **Comprehensive Final Exam in two parts**:
  - **Take-home portion**: *Given in class on Thurs Dec. 4.*
  - **In-class portion**: *Wednesday, Dec. 10, 11:30AM - 1:18 PM* (Closed book, Closed notes). This is a follow-up to the take home portion.
  - Both parts due at the end of the In-Class Portion of the Exam.

**Course Grading:** Grades will be assigned based on clusters in the overall course scores with the following weights:

Home Work Assignments - **25%**
Mid Term Exam - **25%**
Final Exam (Take Home Portion) - **40%**
Final Exam (In Class Portion) - **10%**

# Tentative Course Outline

**I. Introduction (Lecture # 1)**

- General Linear Model-Definition **(Sec. 1.1)**
- General Linear Model-Examples **(Sec. 1.2-1.11)**
- Need for Linear Algebra - Least Squares Estimation

**II. Background Material (Lecture # 2-3)**

- Review of Linear Algebra (**Appendix A.1-A.3) [Reading Assignment prior to Lecture 2]**
- Solving Equations and Generalized Inverses **(Appendix A.4)**
- Constrained Optimization
  - Lagrange Multipliers **(Appendix B)**

**III. The Linear Least Squares problem (Lectures # 4-5)**

- The Normal Equations **(Sec. 2.1)**
- Projections and Solutions to Normal Equations (**Appendix A.5**)
- Geometry of Least Squares **(Sec. 2.2)**
- Reparameterization **(Sec. 2.3)**
- Gram-Schmidt Orthogonalization **(Sec. 2.4) [Reading Assignment]**

**IV. Estimability and Least Squares Estimation (Lectures # 5-6)**

- Non-Full Rank Case **(Sec. 3.1-3.8)**
- Estimation subject to Linear Restrictions **(Sec.3.9)**

**V. Gauss Markov Model (Lectures #7-8)**

- Gauss-Markov Theorem (**Sec. 4.1-4.2**)
- Variance Estimation (**Sec. 4.3**)
- Implications of Model Selection (**Sec. 4.4)**
- Generalized Least Squares **(Sec. 4.5-4.6)**
- Best Estimation of Constrained Parameter Space **(Sec. 4.7)**

**VI. Relevant Distribution Theory (Lectures #9-10)**

- Introduction **(Sec. 5.1)**
- Multivariate Normal Distributions **(Sec. 5.2)**
- Chi-Square, t, F and Related Distributions **(Sec. 5.3)**
- Distributions of Quadratic Forms **(Sec 5.4)**
- Cochran's Theorem (**Sec. 5.5**)

**VII. Inference for the General Linear Model (Lectures #12-15)**

- Distributional Properties of Least Squares Estimates and Residuals **(Sec. 6.1-6.2)**
- Testing General Linear Hypotheses **(Sec. 6.3)**
- LRT and Changes in SSE **(Sec. 6.4-6.5)**
- Identifiability **(Sec. 6.7)**
- Various Issues in Testing Several Hypotheses
  1. Reparemetrization (**Sec. 7.1-7.2**)
  2. Sequence of Nested Hypotheses (**Sec. 7.3**)

3. Orthogonal Polynomials and Contrasts (**Sec. 7.4**)
4. Underfitting, Overfitting and Lack of Fit Test (**Sec. 7.5**)
5. Testing Non-Testable Hypotheses (**Sec. 7.6**)

**VIII. Simultaneous Confidence Intervals and Multiple Comparisons (Lectures # 16-17)**

- Joint and Marginal Confidence Intervals **(Sec. 6.6)**
- Introduction to Multiple Comparison Procedures **(Sec. 6.6)**
    - Scheffe Procedure, Bonferroni t-intervals

**IX. Variance Components and Mixed Models (Lectures #18-19)**

- One-factor random-effects model **(Sec. 8.1-8.2)**
- Mixed-Effects Linear Models **(Sec. 8.3-8.4)**
- Predictions and BLUPs **(Sec. 8.6)**

**Supplementary Reference Books:**


- Arnold, S. F. (1981) *The Theory of Linear Models and Multivariate Analysis,* Wiley
- Graybill, F. A. (1997) Introduction to Matrices with Applications in Statistics, Wadsworth.
- Harville, D. (1997) *Matrix Algebra from a Statistician's Perspective,* Springer-Verlag.
- Jorgensen, B. (1993) *The Theory of Linear Models*, Chapman & Hall.
- McColloch, C.E., S.R. Searle and J.M. Neuhaus (2008) Generalized, Linear and Mixed Models, 2nd Edition, Wiley
- Rao, C. R. and H. Toutenburg (1999) *Linear models: Least Squares and Alternatives,* Springer-Verlag.
- Ravishanker, N. and D. Dey (2002) *A First Course in Linear Model Theory*, Chapman & Hall/CRC
- Rencher, A.C. and G. B. Schaalje (2008) *Linear Models in Statistics*, Wiley
- Seber, G.A.F. (2008) *A Matrix Handbook for Statisticians*, Wiley