Stat 742 | Final Examination (In-Class Portion) | Fall 2007 Dr. Goel

****READ ME****

This in-class portion of the Final Examination consists of four problems, for a total of 60 points. Individual scores for each problem are given on the left margin.

1. [10 points] Consider the Problem 2 on the Take-Home Portion of the Final Examination. Assume that the matrix **X** has full column rank.

   a. [5 points] Does $\tilde{\beta}_1 = \hat{\beta}_1$, imply that $\hat{\beta}_2 = 0$? Explain your answer.

   b. [5 points] Let $\tilde{\beta}_2$ satisfy $\mathbf{X}_2'\mathbf{X}_2\tilde{\beta}_2 = \mathbf{X}_2'\mathbf{Y}$. Does $\tilde{\beta}_1 = \hat{\beta}_1$ imply that $\tilde{\beta}_2 = \hat{\beta}_2$? Explain your answer.

2. [20 points] Consider the Problem 4 on the Take-Home Portion of the Final Examination.

   a. [5 points] Note that $\tau_j, j = 1, 2, \cdots, J$ are not estimable functions for this model. When you perform the F-test to test the hypotheses $H_0 : \tau_j = 0, j = 1, 2, \cdots, J,$ state the hypotheses you are actually testing.

   b. [10 points] In this model, the 2*J* observations can also be transformed into *J* pair-wise differences, and *J* pair-wise sums. Write down the model for these transformed data. Now, show that the Gauss-Markov estimators of any contrast in $\tau_j, j = 1, 2, \cdots, J$ are same as its OLS estimator.

   c. [5 points] Find simultaneous confidence intervals for all possible contrasts in $\tau_j, j = 1, 2, \cdots, J$

3. [5 points] Consider the Problem 5 on the Take-Home Portion of the Final Examination. Provide a test of the hypothesis $H_0 : \eta = 0$ vs. $\eta \neq 0$.

4. [25 points] For each of the following, explain why the statement is *True* or *False*.

   a. [5 points] Let $\hat{Y}$ denote the projection of Y onto the column space of the matrix *X* in a general linear model. The Pearson correlation coefficient between Y and $\hat{Y}$ is always positive.

   b. [5 points] The *F*-test of hypothesis $H_0 : c'\beta = b$ $(b \neq 0)$, for an estimable function $c'\beta$, is same as the *F*-test of hypothesis $H_0^* : kc'\beta = b$ $(k \neq 1)$.

c. [5 points] If the *F*-test rejects the hypotheses $H_0 : \tau_1 = \tau_2 = \cdots = \tau_k$ in a two-way ANOVA model, then there exists at least one significant pair-wise difference according to the Scheffe -Method.

d. [5 points] In a one-factor Random Effect Model, the unbiased estimator of variance components based on the ANOVA method can some times be negative.

e. [5 points] In the simple linear regression model $Y_i = \alpha + \beta(x_i - \bar{x}.) + \varepsilon_i, i = 1, 2, \cdots, n,$ the OLS estimator of α is the projection of the vector $Y_{n\times 1}$ onto the space spanned by the vector $1_{n\times 1}$.