13.

a. \[ 1 = \int_1^\infty \frac{k}{x^3} \, dx \Rightarrow 1 = -\frac{k}{3} x^{-3} \bigg|_1^\infty \Rightarrow 1 = 0 - (-\frac{k}{3})(1) \Rightarrow 1 = \frac{k}{3} \Rightarrow k = 3 \]

b. \[ F(x) = \int_{-\infty}^x f(y) \, dy = \int_1^x 3y^{-4} \, dy = -\frac{3}{3} y^{-3} \bigg|_1^x = -x^{-3} + 1 = 1 - \frac{1}{x^3}. \]

\[ F(x) = \begin{cases} 
0, & x \leq 1 \\
1 - \frac{1}{x^3}, & x > 1 
\end{cases} \]

So

\[ P(x > 2) = 1 - F(2) = 1 - \left( 1 - \frac{1}{2^3} \right) = \frac{1}{2} \text{ or } .125; \]

\[ P(2 < x < 3) = F(3) - F(2) = \left( 1 - \frac{1}{3^3} \right) - \left( 1 - \frac{1}{2^3} \right) = .963 - .875 = .088 \]

c. \[ E(X) = \int_1^\infty x \left( \frac{3}{x^4} \right) \, dx = \int_1^\infty \frac{3}{x^2} \, dx = -\frac{3}{2} x^{-2} \bigg|_1^\infty = 0 + \frac{3}{2} = \frac{3}{2} \]

\[ E(X^2) = \int_1^\infty x^2 \left( \frac{3}{x^4} \right) \, dx = \int_1^\infty \frac{3}{x^2} \, dx = -3x^{-1} \bigg|_1^\infty = 0 + 3 = 3 \]

\[ V(X) = E(X^2) - [E(X)]^2 = 3 - \left( \frac{3}{2} \right)^2 = 3 - \frac{9}{4} = \frac{3}{4} \text{ or } .75 \]

\[ \sigma = \sqrt{V(X)} = \sqrt{\frac{3}{4}} = .866 \]

d. \[ P(1.5 - .866 < x < 1.5 + .866) = P(x < 2.366) = F(2.366) = 1 - (2.366^{-3}) = .9245 \]
Remark: When calculating $P(X < A)$, where $X$ follows a normal distribution with mean=a, std=b, the formula is $P(X < A) = P((X-a)/b < (A-a)/b)$. Since $(X-a)/b$ follows the standard Normal(0,1), $P(X < A) = P(Z < (A-a)/b)$. The same strategy applies to the rest of the problems in this homework.
34. 
   a. \( P(X > .25) = P(Z > -83) = .9987 \)
   b. \( P(X \leq .10) = F(-3.33) = .9984 \)
   c. We want the value of \( t \) that is the 95th percentile (5% of the values are higher). The 95th percentile of the standard normal distribution is 1.645. So \( t = .80 + (1.645)(.06) = .8987 \). The largest 5% of all concentration values are above .3987 mg/cm^3.

38. Let \( X \) denote the diameter of a randomly selected cork made by the first machine, and let \( Y \) be defined analogously for the second machine.
   \[ \begin{align*}
   P(2.9 \leq X \leq 3.1) - P(-1.00 \leq Z \leq 1.00) &= .6826 \\
   P(2.9 \leq Y \leq 3.1) - P(-7.00 \leq Z \leq 3.00) &= .9987
   \end{align*} \]
   So the second machine wins handily.

43. Since 1.28 is the 90th \( z \) percentile (\( z_{.90} = 1.28 \)) and -1.645 is the 5th \( z \) percentile (\( z_{.05} = -1.645 \)), the given information implies that \( \mu + \sigma(1.28) = 10.256 \) and \( \mu - \sigma(-1.645) = 9.671 \), from which \( \sigma(-2.922) = .383 \), \( \sigma = .200 \), and \( \mu = 10 \).

60. 
   a. \( P(X \leq 100) = 1 - e^{-100(.01386)} = 1 - e^{-13.85} = .7499 \)
      \( P(X \leq 200) = 1 - e^{-200(.01386)} = 1 - e^{-2.772} = .9375 \)
      \( P(100 \leq X \leq 200) = P(X \leq 200) - P(X \leq 100) = .9375 - .7499 = .1876 \)
   b. \( \mu = \frac{1}{.01386} = 72.15 \), \( \sigma = 72.15 \)
      \( P(X > \mu + 2\sigma) = P(X > 72.15 + 2(72.15)) = P(X > 216.45) = 1 - \left[1 - e^{-(216.45)(.01386)}\right] = .0498 \)
   c. \( .5 = P(X \leq \mu) \Rightarrow 1 - e^{-(\mu)(.01386)} = .5 \Rightarrow e^{-(\mu)(.01386)} = .5 \)
      \( -\mu(.01386) = \ln(.5) = .693 \Rightarrow \mu = 50 \)

66. 
   a. \( \mu = 20, \sigma^2 = 80 \Rightarrow \alpha \beta = 20, \alpha \beta^2 = 80 \Rightarrow \beta = \frac{80}{20} = 5 \)
   b. \( P(X \leq 24) = F\left(\frac{24}{4},.1\right) = F(6,.1) = .715 \)
   c. \( P(20 \leq X \leq 40) = F(10,.5) - F(5,.5) = .411 \)

Remark: In distribution Gamma(a,b), \( a \) is shape parameter and \( b \) is scale parameter. That is, if \( X \sim \text{Gamma}(a,b) \), then \( \frac{X}{b} \sim \text{Gamma}(a,1) \).