8.

a. \( A_1 \cup A_2 \cup A_3 \)

b. \( A_1 \cap A_2 \cap A_3 \)
c. \[ A'_1 \cap A'_2 \cap A'_3 = A'_1 \cap (A_2 \cup A_3)' \]

\[ \text{Diagram of A_1, A_2, A_3 overlapping regions}\]

d. \[ (A_1 \cap A'_2 \cap A'_3) \cup (A'_1 \cap A_2 \cap A'_3) \cup (A'_1 \cap A'_2 \cap A_3) \]

\[ \text{Diagram of A_1, A_2, A_3 overlapping regions}\]

Note that the above event can also be expressed as

\[ = [A'_1 \cap (A_2 \cup A'_3)] \cup [A_2 \cap (A'_1 \cup A'_3)] \cup [A_3 \cap (A'_1 \cup A_2)'] \]
e. \( A_1 \cup (A_2 \cap A_3) \)

12.

a. \( P(A \cup B) = .50 + .40 - .25 = .65 \)

b. \( P(A \cup B)' = 1 - .65 = .35 \)

c. \( A \cap B' \cup P(A \cap B') = P(A) - P(A \cap B) = .50 - .25 = .25 \)

22.

a. \( P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .4 + .5 - .6 = .3 \)

b. \( P(A_1 \cap A_2') = P(A_1) - P(A_1 \cap A_2) = .4 - .3 = .1 \)

c. \( P(\text{exactly one}) = P(A_1 \cup A_2) - P(A_1 \cap A_2) = .6 - .3 = .3 \)
26.
   a. \( P(A_1') = 1 - P(A_1) = 1 - .12 = .88 \)
   b. \( P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .12 + .07 - .13 = .06 \)
   c. \( P(A_1 \cap A_2 \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) = .06 - .01 = .05 \)
   d. \( P(\text{at most two errors}) = 1 - P(\text{all three types}) \)
      \[ = 1 - \frac{P(A_1 \cap A_2 \cap A_3)}{1} = 1 - .01 = .99 \]

Remark for d: you may calculate \( P(\text{at most two errors}) = P(\text{no error}) + P(1 \text{ error}) + P(2 \text{ errors}) \), but the calculation of each term may be lengthy. Considering the complement of the event is a useful skill in solving many problems, which you will see below.

30.
   a. Because order is important, we'll use \( P_{8,3} = 8(7)(6) = 336 \).
   b. Order doesn't matter here, so we use \( C_{30,6} = 593,775 \).
   c. From each group we choose 2: \( \binom{8}{2} \cdot \binom{10}{2} \cdot \binom{12}{2} = 83,160 \)
   d. The numerator comes from part c and the denominator from part b: \( \frac{83,160}{593,775} = .14 \)
   e. We use the same denominator as in part d. We can have all zinfandel, all merlot, or all cabernet, so \( P(\text{all same}) = P(\text{all z}) + P(\text{all m}) + P(\text{all c}) = \frac{\binom{8}{6} + \binom{10}{6} + \binom{12}{6}}{\binom{30}{6}} = \frac{1162}{593,775} = .002 \)

34
   a. The total number of ways to select a sample of 5 buses from 25 buses is \( \binom{25}{5} = \frac{25!}{5!20!} = 53,130 \)
   b. We first select 4 out of 8 with crack, and then another one out of 17 without crack. Then use the product rule. So the number of ways equals \( \binom{8}{4} \cdot \binom{17}{1} = 1190 \).
c. We’ve calculated the pieces that we need in parts (a) and (b), with part (b) giving us the numerator and part (a) the denominator. Thus, we have

\[
P(\text{exactly 4 have cracks}) = \binom{8}{4} \binom{17}{1} = \frac{1190}{53,130} = 0.022
\]

\[
\binom{4}{1} \binom{25}{1} = \frac{1190}{53,130} = 0.022
\]

d. Now we need to add in the probability that exactly 5 of the 5 sampled buses have cracks.

\[
P(\text{at least 4 have cracks}) = \binom{8}{4} \binom{17}{1} + \binom{8}{5} \binom{17}{0} = 0.022 + 0.001 = 0.023
\]

38,

a. \(P(\text{selecting 2 - 75 watt bulbs}) = \binom{6}{2} \binom{9}{1} = \frac{15 \cdot 9}{455} = .2967\)

\[
\binom{2}{1} \binom{15}{3}
\]

b. \(P(\text{all three are the same}) = \binom{4}{3} + \binom{5}{3} + \binom{6}{3} = \frac{4 + 10 + 20}{455} = .0747\)

\[
\binom{3}{1} \binom{15}{3}
\]

c. \(P = \frac{4 \times 5 \times 6}{455} = \frac{120}{455} = .26\)

Remark: For (a), we first select 2 out of 6 bulbs that are 75W and then select another one from the rest of the nine. For (b), we can have three bulbs that are all 75W, or all 60W, or all 40W. For (c), we select one bulb from each type.
d. To examine exactly one, a 75 watt bulb must be chosen first. (6 ways to accomplish this). To examine exactly two, we must choose another wattage first, then a 75 watt. (9 x 6 ways). Following the pattern, for exactly three, 9 x 8 x 6 ways; for four, 9 x 8 x 7 x 6; for five, 9 x 8 x 7 x 6 x 6.

\[ P(\text{examine at least 6 bulbs}) = 1 - P(\text{examine 5 or less}) \]
\[ = 1 - P(\text{examine exactly 1 or 2 or 3 or 4 or 5}) \]
\[ = 1 - [P(\text{one}) + P(\text{two}) + \ldots + P(\text{five})] \]
\[ = 1 - \left[ \frac{6}{15} + \frac{9 \times 6}{15 \times 14} + \frac{9 \times 8 \times 6}{15 \times 14 \times 13} + \frac{9 \times 8 \times 7 \times 6}{15 \times 14 \times 13 \times 12} + \frac{9 \times 8 \times 7 \times 6 \times 6}{15 \times 14 \times 13 \times 12 \times 11} \right] \]
\[ = 1 - [0.0714 + 0.1582 + 0.2571 + 0.3324 + 0.4187] \]
\[ = 1 - 0.9579 = 0.0421 \]

41.

a. \[ P(\text{at least one F among 1st 3}) = 1 - P(\text{no F's among 1st 3}) \]
\[ = 1 - \frac{4 \times 3 \times 2}{8 \times 7 \times 6} = 1 - \frac{24}{336} = 1 - 0.0714 = 0.9286 \]

An alternative method to calculate \( P(\text{no F's among 1st 3}) \) would be to choose none of the females and 3 of the 4 males, as follows:

\[
\binom{4}{0} \binom{4}{3} = \frac{4}{56} = 0.0714, \text{ obviously producing the same result.}
\]

b. \[ P(\text{all F's among 1st 5}) = \binom{4}{4} \binom{8}{1} = \frac{4}{56} = 0.0714 \]

Remark for b: we need to first pick up all of the 4 F's and then pick up one more M out of the 4 M's

c. \[ P(\text{orderings are different}) = 1 - P(\text{orderings are the same for both semesters}) \]
\[ = 1 - (\# \text{orderings such that the orders are the same each semester})/(\text{total # of possible orderings for 2 semesters}) \]
\[ = 1 - \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)} = 0.99997520 \]

Remark: there are 8! Different orders available, and only 1 of them matches the previous order; all of the rest are not identical to the previous order.