Finite Population of Size $N$

Each element in the population has a label $\{0, 1\}$ [1 = yes, 0 = no]

Draw a random sample of size $n$: Two Schemes

<table>
<thead>
<tr>
<th>Sampling with Replacement</th>
<th>Sampling without Replacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population:</td>
<td>Sample:</td>
</tr>
<tr>
<td>1 5 2 4</td>
<td>2 2</td>
</tr>
<tr>
<td>Sample:</td>
<td></td>
</tr>
<tr>
<td>2 2</td>
<td></td>
</tr>
<tr>
<td>Probability of S in each draw:</td>
<td>Never “remove” selected object from the population</td>
</tr>
<tr>
<td>Distribution:</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

The Hypergeometric Distribution...

...is the exact probability model for the # of $S$’s in a sample of size $n$ when sampling without replacement.

1. Finite population of size $N$
2. Each individual is $S$ (1) or $F$ (0) and there are $M$ total $S$’s in population
3. A random sample of size $n$ is taken without replacement

$X = \#$ of $S$’s in the sample is a Hypergeometric RV:
The Hypergeometric Distribution as product rule:

Sample: \( n \) out of a total of \( N \).

\[
\begin{align*}
M & \quad \text{distinct objects of type 1} \\
N-M & \quad \text{distinct objects of type 2}
\end{align*}
\]

pmf:

\[
\begin{pmatrix}
\end{pmatrix}
\]

Section 3.5

The Negative Binomial Distribution...

Binomial:

Negative Binomial:
The Negative Binomial Distribution...

...is the exact probability model for the “# of failures” needed to observe a predetermined number of successes.

1. Infinite population
2. Sequence of trials: each results in success (S) or failure (F)
3. Trials are independent and identical \(P(S)=p\) remains same
4. Continue till you observe “r” (fixed before hand) success

\[ X = \# \text{ of failures} \] observed till the termination is **Negative Binomial RV**

pmf: