Final review: Practice problems

1. A manufacturer of airplane parts knows from past experience that the probability is 0.8 that an order will be ready for shipment on time, and it is 0.72 that an order will be ready for shipment on time and will also be delivered on time. What is the probability that such an order will be delivered on time given that is was ready on time?

2. The completion of a construction job may be delayed because of a strike. The probabilities are 0.60 that there will be strike, 0.85 that the construction job will be completed on time if there is no strike, and 0.35 that the construction job will be completed on time if there is a strike. What is the probability that the job will be completed? A particular job was completed. What is the probability that there was some disruption due to a strike?

3. Among the 78 doctors on the staff of a hospital, 64 carry malpractice insurance, 36 are surgeons, and 34 are surgeons who carry malpractice insurance. If one of these doctors is chosen by lot to represent the hospital staff at an A.M.A. convention, what is the probability that the one chosen is not a surgeon and does not carry malpractice insurance?

4. The probability of surviving a certain transplant operation is 0.55. If a patient survives the operation, the probability that his or her body will reject the transplant within a month is 0.20. What is the probability of surviving both of these critical stages?

5. Medical records show that one out of 10 persons in a certain town has a thyroid deficiency. If 12 persons in this town are randomly chosen and tested, what is the probability that at least one of them will have a thyroid deficiency?

6. Let X be a random variable such that:

$$p(x) = \begin{cases} 5cx, & \text{for } x = 0, 1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Is X discrete or continuous?
   (b) Find c.
   (c) Find the cumulative distribution function $F(x)$.
   (d) Find $E(X)$ and $Var(X)$.

7. If X had the distribution function

$$F(x) = \begin{cases} 0, & \text{for } x < 1 \\ 1/3, & \text{for } 1 \leq x < 4 \\ 1/2, & \text{for } 4 \leq x < 6 \\ 5/6, & \text{for } 6 \leq x < 10 \\ 1, & x \geq 10 \end{cases}$$

   (a) Is X discrete or continuous?
   (b) $P(2 < X \leq 6)$
   (c) $P(X = 4)$
   (d) the probability distribution of X

8. The probability density function of the random variable X is given by

$$f(x) = \begin{cases} 6x(1-x), & \text{for } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the cumulative distribution function $F(x)$.

(b) Find $E(X)$ and $Var(X)$

(c) $P(X < \frac{1}{4})$ and $P(X > \frac{1}{2})$

9. Let the joint distribution of random variables X and Y be:

$$p(x,y) = \begin{cases} kxy, & \text{for } x = 1,2,3; \quad y = 1,2,3 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find k.

(b) Create a table for the joint probability distribution.

(c) Find the marginal probability mass functions $p_X(x)$ and $p_Y(y)$.

(d) Are X and Y independent?

10. Given the joint probability density function

$$f(x,y) = \begin{cases} \frac{3}{5}x(y+x), & \text{for } 0 < x < 1, 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find $P(0 < X < \frac{1}{2}, 1 < Y < 2)$.

(b) Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$.

(c) Find $E(X)$.

(d) Are X and Y independent?

11. If the values of the joint probability distribution of X and Y are as shown in the table below

$p(x,y)$	$x=0$	$x=1$	$x=2$
$y=0$	1/12	1/6	1/24
$y=1$	1/4	1/4	1/40
$y=2$	1/8	1/20	0
$y=3$	1/120	0	0

(a) $P(X = 1, Y = 2)$

(b) $P(X = 0, 1 \leq Y < 3)$

(c) $P(X + Y \leq 1)$

(d) $P(X > Y)$

12. If the joint density function of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{3}(y+x), & \text{for } 0 < x < 1, 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

find the expected value of $W = 3X + 4Y - 5$.

14. A panel of prospective jurors includes six married men, three single men, seven married women and four single women. If the selection is random, what is the probability that the jury will consist of four married men, one single man, five married women and two single women?

15. An automobile safety engineer claims that 1 ion 10 automobile accidents is due to driver fatigue. What is the probability that at least 3 of 5 independently selected automobile accidents are due to driver fatigue?

17. In a given city, 4 percent of all licensed drivers will be involved in at least one car accident in any given year. What is the probability that among 150 lincesed drivers randomly chosen in this city

    (a) only five will be involved in at least one accident in any given year;

    (b) at most three will be involved in at least one accident in any given year?

18. In the inspection of a fabric produced in continuous rolls, the number of imperfections per yard is a random variable having the Poisson distribution with 0.25 imperfections per yard. Find the probability that 2 yards of the fabric will have at most one imperfection.

19. If a company employs n salespersons, its gross sales in thousands of dollars may be regarded as a random variable having a gamma distribution with $\alpha = 80\sqrt{n}$ and $\beta = 2$. If the sales cost is \$8,000 per salesperson, how many salespersons should the company employ to maximize the expected profit?

20. The mileage (in thousands of miles) that car owners get with a certain kind of radial tire is a random variable having an exponential distribution with $\lambda = 1/40$. Find the probabilities that one of these tires will last

    (a) at least 20,000 miles;

    (b) at most 30,000 miles?

    (c) Sam buys four tires of this kind. What is the probability that at least 2 out of the four last at least 20,000 miles? You may assume that the trials are independent.

21. In a photographic process, the developing time of prints may be looked upon as a random variable having the normal distribution with $\mu = 15.4$ seconds and $\sigma = 0.48$ second. Find the probabilities that the time if takes to develop one of the prints will be

    (a) at least 16.00 seconds
    (b) at most 14.20 seconds
    (c) anywhere from 15.00 to 15.80 seconds
    (d) How long do the quickest 10% of developments take?
    (e) How long do the slowest 10% of developments take?
    (f) What is the probability that exactly 6 out of a random sample of 10 developing times will be between 15.00 and 15.80 seconds?

22. Suppose that the actual amount of instant coffee that a filling machine puts into '6–ounce' jars is a random variable having a normal distribution with $\sigma = 0.05$ ounce. If only 3% of the jars are to contain less than 6 ounces of coffee, what must be the mean fill of these jars?

23. If the number of complaints a dry–cleaning establishment receives per day is a Poisson random variable with $\lambda = 3$, what are the probabilities that it will receive

    (a) two complaints on any given day;
    (b) five complaints altogether on any two given days;
    (c) at least a total of 12 complaints on any three given days?

25. The ages of students at a large university are assumed to be normally distributed with mean 22.3 years and standard deviation 4 years.

    (a) What is the probability that the age of a randomly selected student at the university is greater than 23 years?
    (b) A random sample of 25 students is drawn. What is the probability that the mean age of these studnets is greater than 23 years?
    (c) A random sample of 64 students is drawn. What is the probability that the average age of these studnets is greater than 23 years?
    (d) Do you observe any trend in your answers from (a) through (c)? Why?

Answer Key to Review Problems

1. 0.9 *2.* 0.382 *3.* 12/78 *4.* 0.44 *5.* 0.718

6. (a) Discrete (b) 1/75 (c) 0 for x<1, 1/15 for $1 \leq x < 2$, 3/15 for $2 \leq x < 3$, 6/15 for $3 \leq x < 4$, 10/15 for $4 \leq x < 5$, 1 for $x \geq 5$. (d) E[X]=3.67, V[X]=1.56

7. (a) Discrete (b) ½ (c) 1/6 (d) p(x)=1/3, 1/6, 1/3, 1/6 for x=1,4,6,10 resp'ly

8. (a) F(x) = 0 for x < 0, $3x^2 - 2x^3$ for $0 \leq x \leq 1$, 1 for x>1. (b) E=0.5, V=0.05 (c) 5/32, 0.5

9. (a) 1/36 (b) … easy … (c) $p_X(x) = 1/6, 2/6, 3/6$ for x=1,2,3 similarly for Y. (d) Yes

10. (a) 11/80 (b) $f_X(x) = 6x(x+1)/5$ for 0<x<1, $f_Y(y) = (3y+2)/10$ for 0<y<2 (c) 0.70 (d) No

11. (a) 1/20 (b) 3/8 (c) 0.5 (d) 0.233

12. 1.56 *13.* ?? *14.* 0.045 *15.* 0.00856 *16.* ??

17. (a) 0.014873 (b) 0.1512 *18.* 0.9098 *19.* n=100

20. (a) 0.6065 (b) 0.5276 (c) 0.8282

21. (a) 0.1056 (b) 0.0062 (c) 0.5934 (d) 14.7856 (e) 16.0144 (f) 0.2506 *22.* 6.094

23. (a) 0.224 (b) 0.1606 (c) 0.197 *24.* ?? *25.* (a) 0.4286 (b) 0.1894 (c) 0.0808 (d) ….