Review Questions for Midterm 2

Review Set 1
Question 1 (4 total points)

Let $X$ = the number of programming tasks you complete on any given day at a job. Say that the probability distribution for $X$ is given by the pmf:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$p_X(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
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</tbody>
</table>

Company A proposes to pay you a flat rate of $80 a day.

Company B proposes to pay you $70 on days that you complete 1 task and $100 on days that you complete 2 tasks.

At which company will you make more money in the long run? Justify your answer with a calculation.
Question 2 (18 total points)

Let $X$ = the amount of spillage (in cubic centimeters) that occurs when filling a container of gasoline. Say that for a particular pump at a filling station, the random variable $X$ has probability density function

$$f_X(x) = kx^{-1/2}, \quad 0 \leq x \leq 4.$$ 

(a) (2 pts) Show that $k = 1/4$ makes $f_X(x)$ a legitimate probability density function.

(b) (3 pts) Find the mean of the random variable $X$.

(c) (3 pts) Use the pdf to find the probability that more than 0.25 cubic centimeters of gasoline is spilled.

(d) (3 pts) Find the cumulative distribution function (cdf) of $X$. 

Note: You will need to use the probability you calculated in part (c) to answer parts (e) and (f). If you could not answer part (c) use the value 0.70 instead.

(e) (4 pts) The filling station is interested in how often the pump spills more than 0.25 cubic centimeters of fuel. Say that the pump will be used 15 times on a certain day and that the amount of fuel spilled is independent from usage to usage. Let $Y = $the number of times out of 15 fillings that the spillage is greater than 0.25 cubic centimeters.

What is $P(7 \leq Y \leq 10)$?

(f) (3 pts) Let $Z = $the number of times out of 15 fillings that the spillage is less than 0.25 cubic centimeters. What is the expected value of $Z$?
Question 3 (10 total points)

Cyber attacks are made on an important computer server according to a Poisson process with a rate of 0.10 attacks per minute.

(a) (4 pts) What is the probability that at most 5 attacks occur in a given 20 minute time period?

(b) (3 pts) Let $X =$ the number of attacks that occur in a one hour time period. What is the variance of $X$?

(c) (3 pts) Again let $X =$ the number of attacks that occur in a one hour time period. The cost $Y$ (in dollars) to fix the damage done by the attacks in the one hour time period depends on the number of attacks via $Y = 2X^2$. Find $E(Y)$, the expected cost to fix the attacks that occur in a one hour time period.
**Question 4** (8 total points)

Suppose that $X =$ the amount of residue buildup (in mm) in a small tube and that the probability distribution of $X$ is given by the cumulative distribution function

$$F_X(x) = \begin{cases} 
0 & x < 0, \\
1 - e^{-x^3} & x \geq 0.
\end{cases}$$

(a) (2 pts) What is the probability that there is less than 0.5 mm of residue buildup in a randomly selected tube?

(b) (3 pts) Determine the probability density function (pdf) of $X$.

(c) (3 pts) Find the median of the random variable $X$. 


Review Set 2
1) (12 points) Of all the weld failures in a certain assembly, 20% of them occur in the base metal. A random sample of 20 weld failures is examined.

   a) What is the probability that exactly 5 of them are base metal failures?

b) Calculate the mean number of base metal failures.

c) Calculate the standard deviation of the number of base metal failures.

d) What is the probability the number of base metal failures is within one standard deviation of the mean?
2) (9 points) The number of hits on a certain website follows a Poisson distribution with a mean rate of 4 per minute.
   a) What is the probability that 5 hits are received in a given minute?

   b) How many hits do you expect to receive in 1.5 minutes?

   c) What is the probability that fewer than 3 hits are received in a period of 30 seconds (0.5 minute)?

3) (12 points) The strength of an aluminum alloy is normally distributed with mean 10 gigapascals (GPa) and standard deviation 1.4 GPa.
   a) What is the probability that a specimen of this alloy will have a strength greater than 12 GPa?

   b) What is the probability that a specimen of this alloy will have a strength between 7 and 11 GPa?

   c) Below what strength are the weakest 5% of specimens?
4) (15 points) The concentration of a reactant is a random variable with probability density function

\[ f(x) = \begin{cases} 
  k(x + x^2) & 0 < x < 1 \\
  0 & \text{otherwise}
\end{cases} \]

a) What value of \( k \) makes this a valid probability density function?

b) What is the probability that the concentration is between 0.3 and 0.6?

c) Calculate the mean concentration.

d) Calculate the standard deviation of the concentrations.

e) Find the cumulative distribution function for the concentration.
5) (12 points) One out of every 5000 individuals in a population carries a certain defective gene. A random sample of 1000 individuals is studied. The actual distribution of the number of individuals who carry the gene is binomial, but we can (and will) use the Poisson approximation to the binomial.

a) What is the probability that exactly 1 of the sample individuals carries the gene?

b) What is the probability that more than 2 of the sample individuals carry the gene?

c) Calculate the mean number of sample individuals that carry the gene.

d) Calculate the standard deviation of the number of sample individuals that carry the gene.
These are just the final answers and not “complete solutions.” To get full credits, you will have to show the “complete solutions.”

<table>
<thead>
<tr>
<th>Review SET 1</th>
<th>Review SET 2</th>
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</thead>
<tbody>
<tr>
<td>1 The long-run average amount you earn per day at Company B is more than at Company A, so you will make more money in the long-run with Company A.</td>
<td>1 (X \sim Bin(20, .2)) (a) 0.1746 (b) 4 (c) 1.79 (d) 0.598</td>
</tr>
<tr>
<td>2 (a) (f(x) \geq 0) for all (0 \leq x \leq 4) and verify that (\int_0^4 \frac{4}{x} x^{-1/2} dx = 1) (b) 4/3 (c) 0.75</td>
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<td>(d) (F(x) = \begin{cases} 0 &amp; x &lt; 0 \ \frac{\sqrt{x}}{2} &amp; 0 \leq x \leq 4 \ 1 &amp; x &gt; 4 \end{cases}) (e) 0.31 (f) 3.75</td>
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<tr>
<td>2 (a) (X \sim Poisson (4)); 0.1563 (b) 6 (c) 0.677</td>
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<tr>
<td>3 (a) 0.983 (b) 6 (c) 84</td>
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<td>3 (a) 0.0764 (b) 0.7449 (c) 7.7 GPa</td>
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<td>4 (a) 0.1175 (b) (f(x) = 3x^2e^{-x^3}, x \geq 0), and 0 otherwise. (c) 0.8850</td>
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<td>4 (a) 1.2 (b) 0.2376 (c) 0.7 (d) 0.2236 (e) (F(x) = \begin{cases} 0 &amp; x \leq 0 \ 1.2 \left(\frac{x^2}{2} + \frac{x^3}{3}\right) &amp; 0 &lt; x &lt; 1 \ 1 &amp; x \geq 1 \end{cases})</td>
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<td>5 (a) Poisson approximation with (\lambda = 0.2); 0.1637 (b) 0.0011 (c) 0.2 (d) 0.4472</td>
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