Show all your work for credit and justify your answers. Each problem is worth 5 pts for a total of 100 points possible.

For problems 1-5 use the following information. SET UP THE PROBABILITIES ONLY, DO NOT SOLVE.
An employer has 30 people working on 3 shifts: 1st shift, 2nd shift, and 3rd shift. She has 8 employees on 1st shift, 10 on second shift, and 12 on third shift.

1. She decides to randomly select 3 employees of this company to be the president, vice president, and treasurer of her new special events committee. How many ways are there to do this?

2. Suppose 6 employees are to be randomly selected to win prizes at their company picnic. How many ways are there to do this?

3. Suppose 6 employees are to be randomly selected to win prizes at their company picnic. How many ways are there to randomly choose 6 employees to win prizes, making sure that exactly 2 prizes are given to members of each shift?

4. Suppose 6 employees are to be randomly selected to win prizes at their company picnic. What is the chance that at least one person from each shift wins a prize?

5. Suppose that names are to be selected one by one from the employees until someone from the 1st shift is selected. What is the probability that it is necessary to examine at least 3 names before someone from the first shift is selected? (That means the earliest that someone from the first shift could be selected is on the 3rd trial.)
For problems 6-8 use the following information:
A certain system can experience three different types of defects (where one, two, or all three can happen at the same time). Let $A_i$ denote the event that the system has a defect of type $i$ ($i=1, 2, 3$). Suppose that $P(A_1)=.12$; $P(A_2)=.07$; $P(A_3)=.05$; $P(A_1 \text{ or } A_2)=.13$; $P(A_1 \text{ or } A_3)=.14$; $P(A_2 \text{ or } A_3)=.10$; $P(A_1 \text{ and } A_2 \text{ and } A_3)=.01$.

6. What is the probability that the system does not have a type 1 defect?

7. What is the probability that the system has at most two defects?

8. Given that the system has at most two defects, what is the probability that it has exactly 2 defects?

For Problems 9-11 use the following information:
Plastic bottles are inspected for flaws before shipping. Suppose the proportion of bottles that actually have a flaw is .0002. If a bottle has a flaw, the probability is .995 that it will fail the inspection. If a bottle does not have a flaw, the probability is .99 that it will pass the inspection.

9. If a bottle fails the inspection, what is the probability that it has a flaw?

10. Which of the following is the more correct interpretation of the answer to part a?
   (Check the best answer)
   1. Most bottles that fail inspection do not have a flaw._____  
   2. Most bottles that pass inspection do have a flaw______  
   3. Both 1 and 2 have equal probability_____

11. What percentage of the bottles fail inspection overall?
For problems 12-15 use the following information:

\[
F(x) = \begin{cases} 
0 & x < 1 \\
.3 & 1 \leq x < 3 \\
.4 & 3 \leq x < 4 \\
.45 & 4 \leq x < 6 \\
.6 & 6 \leq x < 12 \\
1 & x \geq 12
\end{cases}
\]

12. Graph the CDF.

13. Find \( P(X > 4) \) using the CDF

14. Find \( p(x) \)

15. What is the probability that \( X \) equals 6?
For problems 16-20 use the following information:
Specifications call for the thickness of aluminum sheets that are to be made into
cans to be between 8 and 11 thousands of an inch. Let X be the actual thickness of
an aluminum sheet that is manufactured. Assume the probability density function of
X is given by: \( f(x) = \frac{x}{c} \), for \( 6 < x < 12 \), and 0 otherwise, where x is in thousands of an
inch and c is some constant.

16. Is X a continuous or discrete random variable? Justify your answer.

17. Find the value of c that makes this a legitimate probability density function.

18. What proportion of sheets will meet the specifications?

19. Ninety percent of the sheets manufactured are thinner than what value?

20. What is the probability that \( X = 10 \)?