

STAT 693: Problem Set #2

Problem 1: (Johnson and Wichern 2.10) Consider the matrices

$$\mathbf{A} = \begin{bmatrix} 4 & 4.001 \\ 4.001 & 4.002 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 4 & 4.001 \\ 4.001 & 4.002001 \end{bmatrix}$$

These matrices are identical except for a small difference in the (2,2) position. Moreover, the columns of \mathbf{A} (and \mathbf{B}) are nearly linearly dependent. Show that $\mathbf{A}^{-1} \approx 3\mathbf{B}^{-1}$. Consequently, small changes—perhaps caused by rounding—can give substantially different inverses.

Problem 2: Give a convincing argument (though not necessarily a formal proof) that the determinant of any $n \times n$ upper triangular matrix is the product of the elements on the main diagonal.

Problem 3: Find the inverse of

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & -1 \\ 0 & -2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

both by cofactor and the triangular methods.