

## STAT 693: Problem Set #1

Problem 1: Find a basis for the vector space defined by

$$\{\mathbf{v} = (x, y, z, w) \in \mathfrak{R}^4 : 3x - y - z + w = 0\}$$

and determine its dimension.

Problem 2: In  $\mathfrak{R}^2$ , write the vector  $\mathbf{x} = [2 \ -1]'$  in terms of the basis vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Problem 3: Let  $H$  be the vector space defined as

$$H = \{(x, y, z) \in \mathfrak{R}^3 | x - y - z = 0\}$$

and let  $\mathbf{v} = [-1 \ 2 \ 4]'$ . Then do the following:

- Find an orthonormal basis for  $H$ .
- Find  $\text{proj}_H \mathbf{v}$ .
- Find an orthonormal basis for  $H^\perp$ .
- Write  $\mathbf{v}$  as  $\mathbf{h} + \mathbf{p}$ , where  $\mathbf{h} \in H$  and  $\mathbf{p} \in H^\perp$ .

Problem 4: Determine whether the following vectors constitute a linearly independent set.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ -1 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} -3 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$