
Finding Binomial Probabilities

Assume $X \sim B(n,p)$, if $k \leq n$ then

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. Note: $a! = a(a-1)(a-2)\dots 1$ and $0! = 1$.

=> This is the **binomial formula**.

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Example: Free-Throws (Lecture 16)

X is the number of shots made in three attempts:

x	P(X=x)
0	0.08
1	0.1+0.1+0.1=0.3
2	0.14+0.14+0.14 = 0.42
3	0.2

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$X \sim B(3, 0.58)$ so we can use the binomial formula to get the probability distribution of X. For example,

$$P(X = 2) = \binom{3}{2} (0.58)^2 (1-0.58)^1 = 0.42$$

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Finding Binomial Probabilities Using Minitab:

Imagine that we are interested in knowing the probability that a 0.58 percent shooter makes 10 out 20 free-throws.

1) $P(X = 10) = \binom{20}{10} (0.58)^{10} (1-0.58)^{20-10}$

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2) Using Minitab

- Calc - Probability Distributions - Binomial
- Check **Probability**
- Number of Trials: **20**
- Probability of Success: **0.58**
- Check **Input Constant: 10**

=> $P(X=10) = 0.1359$

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Using Minitab to find cumulative probabilities,
e.g. $P(X < 8) = P(X \leq 7)$

Instead of finding $P(X \leq 7) = P(X=0)+P(X=1)+\dots+P(X=7)$ directly, use the cumulative probability feature:

- Calc - Probability Distributions - Binomial
- Check **Cumulative Probability** (instead of **Probability**)
- Number of Trials: **20**
- Probability of Success: **0.58**
- Check **Input Constant: 7**

=> $P(X < 8) = 0.0324$

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Binomial Mean and Standard Deviation

If $X \sim B(n,p)$, what are μ_X and σ_X ?

X is the number of successes in n independent observations that each have probability of success p .

Let S_i be the random variable that indicates whether the i^{th} observation was a success ($S_i = 1$) or a failure ($S_i = 0$). The distribution of each S_i is

Outcome	1	0
Probability	p	$1-p$

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Using the definitions of the mean and variance of discrete random variables,

$$\mu_S = (1)(p) + (0)(1-p) = p$$

$$\sigma_S^2 = (1-p)^2p + (0-p)^2(1-p) = p(1-p)$$

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Since $X = S_1 + S_2 + \dots + S_n$, then

$$\mu_X = np$$

$$\sigma_X^2 = np(1-p) \text{ or } \sigma_X = \sqrt{np(1-p)}$$

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What about the sample proportion?

$$\hat{p} = X/n$$

Using the rule for linear functions of random variables,

- $\mu_{\hat{p}} = p$
- $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

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Question: What does the sampling distribution of X look like?

Use Minitab generate lots of samples from a $B(20,0.58)$ distribution:

- Calc - Random Data - Binomial
- Generate **100** rows of data
- Store in Columns **C1**
- Number of Trials: **20**
- Probability of Success: **0.58**

Look at the histogram of C1.

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Question: What does the sampling distribution of \hat{p} look like?

Create a new column with the proportions:

- Calc - Calculator
- Store result in: **C2**
- Expression: **C1/20**

Look at the histogram of C2.

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Normal Approximation for Counts and Proportions

- X is approximately $N(np, \sqrt{np(1-p)})$.
- \hat{p} is approximately $N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$.

Rule of Thumb: This approximation is only valid for values of n and p that satisfy $np \geq 10$ and $n(1-p) \geq 10$.