Finding Binomial Probabilities

Assume X ~ B(n,p), if k <= n then

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. Note: a! = a(a-1)(a-2)···1 and 0! = 1.

=> This is the **binomial formula**.

Example: Free-Throws (Lecture 16)

X is the number of shots made in three attempts:

x	P(X=x)
0	0.08
1	0.1+0.1+0.1=0.3
2	0.14+0.14+0.14 = 0.42
3	0.2

X ~ B(3, 0.58) so we can use the binomial formula to get the probability distribution of X. For example,

$$P(X = 2) = \binom{3}{2}(0.58)^2(1-0.58)^1 = 0.42$$

Finding Binomial Probabilities Using Minitab:

Imagine that we are interested in knowing the probability that a 0.58 percent shooter makes 10 out 20 free-throws.

1) $P(X = 10) = \binom{20}{10}(0.58)^{10}(1-0.58)^{20-10}$

2) Using Minitab
- **Calc - Probability Distributions - Binomial**
- Check **Probability**
- Number of Trials: **20**
- Probability of Success: **0.58**
- Check **Input Constant: 10**

=> P(X=10) = 0.1359

Using Minitab to find cumulative probabilities, e.g. P(X < 8) = P(X <= 7)

Instead of finding P(X<=7) = P(X=0)+P(X=1)+···+P(X=7) directly, use the cumulative probability feature:
- **Calc - Probability Distributions - Binomial**
- Check **Cumulative Probability** (instead of **Probability**)
- Number of Trials: **20**
- Probability of Success: **0.58**
- Check **Input Constant: 7**

=> P(X<8) = 0.0324

Binomial Mean and Standard Deviation

If X ~ B(n,p), what are μ_X and σ_X?

X is the number of successes in n independent observations that each have probability of success p.

Let S_i be the random variable that indicates whether the i^{th} observation was a success ($S_i = 1$) or a failure ($S_i = 0$). The distribution of each S_i is

Outcome	1	0
Probability	p	1-p

Using the definitions of the mean and variance of discrete random variables,

$$\mu_S = (1)(p) + (0)(1-p) = p$$

$$\sigma^2_S = (1-p)^2 p + (0-p)^2 (1-p) = p\,(1-p)$$

Since $X = S_1 + S_2 + \cdots + S_n$, then

$\mu_X = np$

$\sigma^2_X = np(1-p)$ or $\sigma_X = \sqrt{np(1-p)}$

What about the sample proportion?

$\hat{p} = X/n$

Using the rule for linear functions of random variables,

- $\mu_{\hat{p}} = p$
- $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

Question: What does the sampling distribution of X look like?

Use Minitab generate lots of samples from a B(20,0.58) distribution:

- **Calc - Random Data - Binomial**
- Generate **100** rows of data
- Store in Columns **C1**
- Number of Trials: **20**
- Probability of Success: **0.58**

Look at the histogram of C1.

Question: What does the sampling distribution of $\hat{p}$ look like?

Create a new column with the proportions:

- **Calc - Calculator**
- Store result in: **C2**
- Expression: **C1/20**

Look at the histogram of C2.

Normal Approximation for Counts and Proportions

- X is approximately $N(np, \sqrt{np(1-p)})$.
- $\hat{p}$ is approximately $N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$.

Rule of Thumb: This approximation is only valid for values of n and p that satisfy np >= 10 and n(1-p) >= 10.