Rules for Means of Random Variables

Rule 1

If X is a random variable and a and b are fixed numbers, then

$$\mu_{a+bx} = a+b\mu_X$$

Rule 2

If X and Y are random variables, then

$$\mu_{X+Y} = \mu_X+\mu_y$$

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Variance of a Discrete Random Variable

Suppose that X is a discrete random variable whose distribution is

Value of X	x_1	x_2	x_3	···	x_k
Probability	p_1	p_2	p_3	···	p_k

and that μ_X is the mean of X. The **variance** of X is

$$\sigma^2_X = (x_1-\mu_X)^2p_1 + (x_2-\mu_X)^2p_2+\cdots+(x-\mu_X)^2p_k$$

$$\Sigma(x_i-\mu_X)^2p_i$$

The **standard deviation** of X is the square root of the variance.

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Example: Free-Throw (Lecture 16)

What is the standard deviation of shots made by a 58% shooter who shoots three shots?

The probability distribution for X (number of shots made):

x	P(X=x)
0	0.08
1	0.1+0.1+0.1=0.3
2	0.14+0.14+0.14 = 0.42
3	0.2

From lecture 17, $\mu_X = 1.74$

$$\sigma^2_X = (0-1.74)^2\, P(X=0) + (1-1.74)^2\, P(X=1) + (2-1.74)^2\, P(X=2) + (3-1.74)^2\, P(X=3)$$
$$= (3.0276)(0.08)+(0.5476)(0.3)+(0.0676)(0.42)+(1.5876)(0.2)$$
$$= 0.7524$$

$\sigma_X = \text{sqrt}(\sigma^2_X) = 0.8674$

Rules for Variances

Rule 1

If X is a random variable and a and b are fixed numbers, then

$$\sigma^2_{a+bX} = b^2 \sigma^2_X$$

Rule 2

If X and Y are *independent* random variables, then

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$$
$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$$

This is the **addition rule for variances of independent random variables.**

Rule 3

If X and Y are dependent and have correlation ρ, then

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y - \rho\sigma_X\sigma_Y$$
$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y - \rho\sigma_X\sigma_Y$$

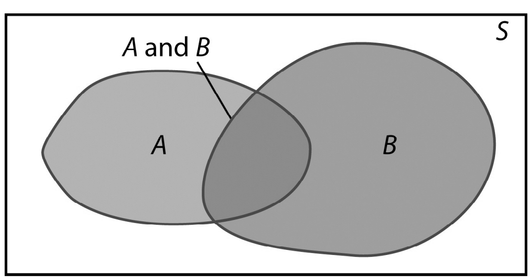
This is the **general rule for variances of random variables**.

General Probability Rules

General Addition Rule for Unions of Two Events

For any two events A and B,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



What is P(A and B)?

- If A and B are *independent*, then by the multiplication rule

  P(A and B) = P(A)P(B)

- If A and B are *dependent,* then we need a general multiplication rule:

  P(A and B) = P(A)P(B|A)

  where P(B|A) is the *conditional probability* that B occurs given the information that A occurs.

- Definition of **independence**: Two events A and B are independent if

  P(A|B) = P(A) and P(B|A) = P(B).

  So our old multiplication rule for independent events is a special case of the general multiplication rule.

- Definition of **conditional probability**: when P(A) > 0, the conditional probability of B given A is

  P(B|A) = P(A and B) / P(A)