Statistics 528 – Homework 7 Solutions

5.2
(a) No, not a fixed number of trials.
(b) Yes, we choose a sample of fixed size and since 100 persons are chosen at random, each person in the sample has the same probability of opposing the death penalty ran on sentence (the proportion of adult residents in the city opposing the death penalty). Furthermore, sampling 100 persons from a large city makes it reasonable to assume that outcomes (oppose or not oppose) are independent.
(c) Yes, we have a fixed number of trials (52 weeks), outcomes (win or lose) are independent due to the random selection of winning number, and the probability of winning is the same each week.

5.6
(a) Use table C. If 80% graduate, then 20% fail to graduate. Probability that 11 graduate is the same as the probability that 9 fail to graduate. This is 0.0074.
(b) Having 11 or fewer graduate is the same as 9 or more failing to graduate. This is 0.0074+0.0020+0.0005+0.0001=0.01

5.8
(a) mean =20*0.8=16.
(b) standard deviation=square root(20*0.8*0.2)=1.789.
(c) With p=0.9, standard deviation=square root(20*0.9*0.1)=1.3416; with p=0.99, , standard deviation=square root(20*0.99*0.01)=0.4450. This shows that the standard deviation decreases as p gets closer to 1.

5.14
(a) Since np=1000*0.29=290 and n(1-p)=1000*0.71=710, both are much larger than 10, the distribution of the sample proportion is approximately Normal distribution with the mean 0.29 and the standard deviation 0.0143(=square root (0.29*0.71/1000)).
For 26%, z = (0.26-0.29) / 0.0143 = -2.09 and for 32%, z = (0.32 - 0.29) / 0.0143= 2.09.
Therefore, P(0.26< $\hat{p}$ <0.32) =P(-2.09<Z<2.09)=0.9634.
(b) For n=250, the standard deviation of $\hat{p}$ is 0.0287(=square root (0.29*0.71/250)).
P(-0.03/0.0287<Z<0.03/0.0287)=P(-1.045<Z<1.045)=0.7041.
For n=4000, the standard deviation of $\hat{p}$ is 0.00717(=square root (0.29*0.71/4000)).
So, P(-0.03/0.00717<Z<0.03/0.00717)= P(-4.181<Z<4.181) $\approx$ 1. The probability of being within 3% of the truth increases as n increases due to the reduction of standard deviation.

5.28
(a) 10/square root(3)=5.7735
(b) To reduce the standard deviation by a factor of 2 requires 4(2^2) times as much data, so n=4. The average of several observations is less variable than a single observation.

5.33

(a) Summing the fourth row gives the mean to be 2.45. Summing the bottom row and taking the square root gives the standard deviation 1.099.

Grade	A	B	C	D	F
Prob	0.18	0.32	0.34	0.09	0.07
X	4	3	2	1	0
X*prob	0.72	0.96	0.68	0.09	0
x-mean	1.55	0.55	-0.45	-1.45	-2.45
$(\text{x-mean})^2$	2.4025	0.3025	0.2025	2.1025	6.0025
$(\text{x-mean})^2$*prob	0.4325	0.0968	0.0688	0.1892	0.4202

(b) The mean of 50 students is the same as the mean of the distribution of grades, 2.45. The standard deviation of the sample mean is 1.099/square root(50)=0.1554.

(c) For a single student P(X>=3)=P(X=3)+P(X=4)=0.18+0.32=0.50.
For the average of 50 students, $\bar{X}$ approximately follows Normal distribution with the mean 2.45 and the standard deviation 0.1554 by the Central Limit Theorem.
P($\bar{X}$ >=3)=P(Z>(3-2.45)/0.1554) = P(Z>=3.54)=0.0002.

5.38

(a) The average will have the normal distribution with mean 55,000 miles and standard deviation 4,500/square root(8)=1590.99 by the property of normal distributions.
(b) The z-score of 51,800 is (51800-55000)/1590.99=2.011. Thus,
P($\bar{X} \leq$ 51,800)= P(Z $\leq$ -2.011)=0.0221.

5.44

(a) The total resistance of two components, each with normal distribution follows a normal distribution by the property that any linear combination of normal random variables is normally distributed. The mean of the total is the sum of the means, 100+250=350. The standard deviation is square root($2.5^2+2.8^2$)=3.7537 by the independence of the two components. So, the total resistance has N(350,3.7537).
(b) Let X be the total resistance. Then,
P(345<X<355)=P((345-350)/3.7537<Z<(355-350)/3.7537)=0.8171.