Statistics 528 – Homework 6 Solutions

4.10
(a) The sample space would be {female, male}.
(b) The sample space would be {6, 7, …, 20}.
(c) The sample space would be the interval [2.5, 6]/min.
(d) For living human beings, heart rate is probably in the range of 40-200 and would be a whole number. The theoretical range would be positive whole numbers.

4.14
The probability they both have type O is the product of the probabilities for each country, because two persons are independently chosen. This is 0.45*0.35=0.1575.
To find the probability they have the same blood type requires doing the above calculation for each type. Then, because the types are disjoint, we can add the results. The probability they both have type A is 0.40*0.27=0.108.
Both type B :0.11*0.26=0.0286. Both type AB: 0.04*0.12=0.0048. Therefore,
P(both have the same blood type)
= P(both have type O)+P(both have type A)+P(both have type B)
+P(both have type AB)
= 0.1575+0.108+0.0286+0.0048=0.2928.

4.20
(a) Yes, the total probability (=0.000+0.003+0.060+0.062+0.036+0.121+0.691+0.027) is 1 and all the entries are nonnegative.
(b) P(A) is the total of the Hispanic column, 0.125.
(c) B^c is the event that the chosen person is non-white.
P(B)=0.060+0.691=0.751, so $P(B^c)=1-0.751=0.249$.
(d) The event is $A^c \cap B$. The probability is 0.691 directly from the table.
Note that unless two events A^c and B are independent, $P(A^c \text{ and } B) \neq P(A^c)\, P(B)$. In fact, $P(A^c \text{ and } B) = 0.691$ while $P(A^c)\, P(B) = (1-0.125)(0.751) = 0.6571$. This implies that they are not independent.

4.30
The probability that a randomly chosen person is not a universal donor is 1-0.07=0.93.
Since 10 individuals appearing at random are independent, the probability that none of them is O-negative is $0.93^{10}=0.484$.
P(at least one O-negative) =1-P(none of them is O-negative)= 1-0.484=0.516.

4.42
(a) BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG.
Because the chance that a couple would have a girl is the same as that of a boy, and the sexes of three children are independent of each other, each has probability 1/8.
(b) There are 3 combinations with exactly two girls, so P(X=2)=3/8.
(c)

X	0	1	2	3
Probability	1/8	3/8	3/8	1/8

4.52

(a) For the uniform density, the probability of an interval is proportional to its length since the area under the density curve over an interval is 1* the length of the interval. The interval (0.27, 1) has length 0.73, so $P(X \geq 0.27)=0.73$.

(b) 0 because a line has area 0.

(c) This is the same as (a), since there is no density to the right of 1.

(d) These are disjoint intervals, so we can add the probability of each one. Each is of length 0.1, so the total probability is 0.2.

(e) $P(X>0.8 \text{ or } X<0.3)=1-P(0.3 \leq X \leq 0.8) = 1-0.5=0.5$.

4.54

(a) The area of a triangle is one half of the base length (b) times the height (h). For this triangle, b=2 and h is specified as 1. This gives area=1.

(b) $P(Y<1)$ is the area of the left half of the triangle, so the probability is 1/2.

(c) $P(Y<0.5)$ is the area of a triangle with b=0.5, and h= 0.5. The probability is 1/8.

4.56

(a) $P(\hat{p}>0.16)=P(Z>(0.16-0.15)/0.0092)=P(Z>1.09)=0.1379$.

(b) $P(0.14<\hat{p}<0.16)=P(-1.09<Z<1.09)=0.8621-0.1379=0.7242$.

4.74

(a) μ_X =540*0.1+545*0.25+550*0.3+555*0.25+560*0.1=550,

σ_X^2 =(540-550)^2*0.1+(545-550)^2*0.25+(550-550)^2*0.3+(555-550)^2*0.25 +(560-550)^2*0.1 =32.5, so $\sigma_X = \sqrt{32.5}$ =5.701.

(b) $\mu_{X-550} = \mu_X - 550$ =550-550=0°C, and

$\sigma_{X-550} = \sigma_X$ =5.701°C since shifting a random variable by a constant would not change its standard deviation.

(c) $\mu_Y = \frac{9}{5}\mu_X + 32$ =1022°F, and $\sigma_Y = \frac{9}{5}\sigma_X$ =10.26°F.

4.75

(a)The mean of the difference is the difference of the means or

$\mu_{Y-X} = \mu_Y - \mu_X$ = 2.001-2.000 = 0.001g.

$\sigma_{Y-X}^2 = \sigma_Y^2 + \sigma_X^2 = 0.001^2 + 0.002^2$ =0.000005 by the independence of X and Y.

The standard deviation $\sigma_{Y-X} = \sqrt{0.000005}$ = 0.002236g.

(b) The mean of the average is the average of the means or

$\mu_Z = \mu_{(X+Y)/2} = (\mu_X + \mu_Y)/2$ = 2.0005g.

$\sigma_Z^2 = \sigma_{(X+Y)/2}^2 = (\frac{1}{2})^2 \sigma_{X+Y}^2 = \frac{1}{4}(\sigma_X^2 + \sigma_Y^2)$ = 0.00000125. So, σ_Z = 0.001118g.

Z is slightly more variable than Y, because it has a slightly larger standard deviation.

4.94

(a) P(female)=922/1654=0.557

(b) P(female/person chosen received a professional degree)
=P(female and professional)/P(professional)=(32/1654)/(72/1654)=0.444

(c) Not independent, since P(female/professional) is not equal to P(female).