2. (a) Unbiased means: \( E(\hat{\theta}) = \theta \). Thus the distribution of \( \hat{\theta} \), which is random, is centered at \( \theta \), the parameter whose value we want to estimate.

(b) Compare \( \hat{\theta} \) and \( \hat{\theta}_2 \) on the basis of their variances. We want the one with the smaller variance.

(c) \( E(\hat{\mu}) = E\left[\frac{1}{3}(\sum_{i=1}^{3} X_i)\right] = \frac{1}{3}(\sum_{i=1}^{3} E(X_i)) \). Since \( \mu \) is the “population mean,” \( E(X_i) = \mu \, (i=1,2,3) \). Thus, \( E(\hat{\mu}) = \frac{1}{3}(3\mu) = \mu \).

3. (a) To be within 5%, we want the half-width to be 5%. This means means \( w = 0.10 \). Use the approximate formula on p.296,

\[
 n \approx \frac{4z^2 \hat{p} \hat{q}}{w^2}. 
\]

Since we want 90% confidence, we use \( z_{\frac{\alpha}{2}} = z_{0.05} = 1.645 \). Since we don’t know \( \hat{p} \) or \( \hat{q} \), we replace them with the conservative values \( \hat{p} = \hat{q} = \frac{1}{2} \). Thus,

\[
 n \approx \left( \frac{1.645}{0.10} \right)^2 = 270.6.
\]

Use \( n = 271 \).

(b) \( n = 200, \hat{p} = \frac{80}{200} = 0.4 \)

For 92% confidence, use \( z_{0.04} \), with area 0.96 below it. \( z_{0.04} = 1.75 \).

Approximate C.I. is

\[
 \hat{p} \pm \frac{z}{\sqrt{n}} \sqrt{\hat{p}\hat{q}}.
\]

This is \( 0.4 \pm \left( \frac{1.75}{\sqrt{200}} \right) \sqrt{0.24} = 0.4 \pm 0.06 = (0.34, 0.46) \).

4. (a) Since \( \bar{x} \) is in the center of the interval,

\[
 \bar{x} = \frac{4.216 + 5.784}{2} = 5.0
\]
(b) The C.I. is $5 \pm \frac{z(\sigma)}{\sqrt{n}} = 5 \pm \frac{z(4)}{12} = 5 \pm \frac{2.352}{3}$. From (a) we know the C.I. $= 5 \pm 0.784$. Thus, we see that $z = 2.352$. This is $z_{\frac{a}{2}}$, so we look in Table A.3 to see that the area below 2.352 is 0.9906. Thus, the area above 2.352 is 0.0094, and thus $\frac{a}{2} = 0.0094$, and $\alpha = 0.0188$.

The confidence is $1 - 0.0188 = 0.98$.

(c) Since $\sigma^2$ is given as 16, use $\sigma = 4$. For 95% confidence, $z = z_{\frac{a}{2}} = z_{0.025} = 1.96$. Within 2 points means that the half-width,

$$w = 2 = \frac{z\sigma}{\sqrt{n}} = \frac{(1.96)4}{\sqrt{n}}.$$

Thus

$$\sqrt{n} = \frac{(1.96)4}{2} = 3.92.$$ Then, $n = 15.37$ and we use $n = 16$.

5. Let $\mu$ be true average degree of polymerization.

$n = 17$, so, $\nu = (n - 1) = 16$.

The critical value for a 95% interval is $t_{0.025,16} = 2.120$. The interval is thus

$$438.29 \pm (2.120)[\frac{15.14}{\sqrt{17}}] = 438.29 \pm 7.78 = (430.51, 446.07).$$

Since 440 is in the confidence interval, it is a plausible value for the true average value, $\mu$.

Since 450 is not in the confidence interval, it is not a plausible value for the true average value, $\mu$. 