Sta428: Answers, Sample Midterm 2

1. (a) Let \( \mu \) be true average mileage.

\[
H_0: \ \mu = 32, \quad H_a: \ \mu > 32
\]

Assume the normal distribution. \( n=6, \ \bar{x} = 32.92 \)

\[
Z = \frac{32.92 - 32}{1.5/\sqrt{6}} = \frac{0.92}{0.6124} = 1.50
\]

Reject if \( Z > z_{0.05} = 1.645 \)

Since \( 1.50 < 1.645 \), do not reject \( H_0 \).

(b) Since you failed to reject \( H_0 \), you could have made a type II error.

(c) The rejection region is:

Reject if

\[
\bar{X} > 32 + (1.645)(0.6124) = 33.007
\]

Thus, \( \beta(34) = P(\text{Accept} | \mu = 34) = P\left(\frac{\bar{X} - 34}{0.6124} < \frac{32 - 34}{0.6124} + \frac{(1.645)(0.6124)}{0.6124}\right) \)

\[
= P(Z < -3.26 + 1.645) = P(Z < -1.62) = 0.053
\]

(d) Sample size to make type II error probability = 0.025 at 34.

Thus for general \( n \), \( \beta(34) = P\left(\frac{\bar{X} - 34}{1.5/\sqrt{n}} < \frac{\sqrt{n}(32 - 34)}{1.5} + 1.645\right) \)

\[
= P(Z < -1.33\sqrt{n} + 1.645) = P(Z < -1.96) = 0.025
\]

Hence,

\[
-1.33\sqrt{n} + 1.645 = -1.96 \quad \text{or} \quad \sqrt{n} = \frac{1.96 + 1.645}{1.33} = 2.704 \quad \text{or} \quad n = 7.31
\]

Use, \( n = 8 \).

2. (a) \( p = P(\text{voter opposed}) \)

\[
\hat{p} = \frac{465}{900} = 0.5167
\]

\[
Z = \frac{\hat{p} - 0.5}{\sqrt{(0.5)(0.5)/900}} = \frac{0.0167}{0.0167} = 1
\]

Reject if \( Z > z_{0.05} = 1.645 \). Thus, we DO NOT reject. There is not strong evidence that a majority of voters oppose the lottery.
(b) You might be making a type II error (failing to reject \( H_0 \) when it is false). The consequence would be adoption of the lottery when a majority of voters oppose it.

(c) We want \( P(\text{Reject } H_0 \text{ when } p = 0.53) = 1 - \beta(0.53) = P(\hat{p} > 0.5 + 1.645(0.0167)) = P(\hat{p} > 0.52747) \)

\[
P(\hat{p} > 0.5 + 1.645(0.0167)) = P(\hat{p} > 0.52747) = P\left(\frac{\hat{p} - 0.53}{\sqrt{\frac{(0.53)(0.47)}{900}}} > \frac{0.52747 - 0.53}{\sqrt{\frac{(0.53)(0.47)}{900}}}\right) = P(Z > -0.00253)
\]

\[
P(Z > -0.1521) = 0.56
\]

(d) \( \beta(0.53) = P(\hat{p} < 0.5 + 1.645\sqrt{\frac{0.5(0.5)}{n}}) \)

\[
P(\hat{p} < 0.5 + 1.645\sqrt{\frac{0.5(0.5)}{n}}) = P\left(\frac{\hat{p} - 0.53}{\sqrt{\frac{(0.53)(0.47)}{n}}} < \frac{0.5 - 0.53}{\sqrt{\frac{(0.53)(0.47)}{n}}} + 1.645\sqrt{\frac{0.5(0.5)}{n}}\right) = P(Z < -0.03\sqrt{n} + 1.645 \frac{0.5}{0.4991})
\]

\[
P(Z < -0.0601\sqrt{n} + 1.648) = 0.1 = P(Z < -1.282)
\]

Thus, we want

\[-0.0601\sqrt{n} + 1.648 = -1.282, \text{ or } \sqrt{n} = \frac{2.93}{0.0601} = 48.75 \Rightarrow n = 2376.8 \]

Use \( n = 2377 \).

3.

\[
\bar{x}_C = 47, \quad \bar{x}_T = 55, \quad (\bar{x}_C - \bar{x}_T) = -8
\]

\[
\sqrt{\frac{(27.053)^2 + (27.1956)^2}{6}} = \sqrt{245.2442} = 15.66
\]

\[
T = \frac{-8}{15.66} = -0.51
\]

The 2-sided \( T \) test rejects if \( |T| > t_{0.025,v} \)

Now to calculate \( v \). For the numerator, we need \( (245.2442)^2 = 60,144.74 \).

For the denominator we need:

\[
\frac{S_1^4 + S_2^4}{m^2(m-1)} = \frac{(27.053)^4 + (27.1956)^4}{5(36)}
\]

\[
= \frac{535,626.099 + 547,009.14}{180} = 6,014.6402
\]

Thus,
\[ \nu = \frac{60,144.74}{6,014.6402} = 9.9997 \]

use \( \nu = 9 \). \( t_{0.025,9} = 2.262 \)

DO NOT REJECT.

(b). Let \( D = (X_C - X_T) \). The observed \( D \)’s are: 8, 2, 13, 13, 6, 6. \( \sum D_i = 48 \), \( D = 8 \), \( S_D = 4.34 \).

The point estimate of \( (\mu_C - \mu_T) \) is 8. The Confidence Interval for \( (\mu_C - \mu_T) \) is

\[ \bar{D} \pm t_{.025,5} \left( \frac{S_D}{\sqrt{6}} \right). \text{Since } t_{.025,5} = 2.571, \text{this gives} \]

\[ 8 \pm 2.571 \left( \frac{4.34}{\sqrt{6}} \right) = 8 \pm 4.55 = (3.45,12.55) \]

Since this interval is well away from zero, the paired comparison approach shows (at the 95% level) a significant difference from zero and suggests strongly that the reaction time for the group with the regular display is larger than that for the group that had the special display.

4. (Prob. 47). \( H_0 \) will be rejected if \( z \leq -z_{.01} = -2.33 \). With \( \hat{p}_1 = .150 \), and \( \hat{p}_2 = .300 \), \( \hat{p} = \frac{30 + 80}{200 + 600} = \frac{210}{800} = .263 \), and \( \hat{q} = .737 \). The calculated test statistic is

\[ z = \frac{.150 - .300}{\sqrt{(263)(.737)\left(\frac{1}{200} + \frac{1}{600}\right)}} = \frac{- .150}{.0359} = -4.18. \]

Because \( -4.18 \leq -2.33 \), \( H_0 \) is rejected; the proportion of those who repeat after inducement appears lower than those who repeat after no inducement.

5. (Prob. 59) We test \( H_0: \sigma_1^2 = \sigma_2^2 \) vs. \( H_a: \sigma_1^2 \neq \sigma_2^2 \). The calculated test statistic is

\[ f = \frac{(2.75)^2}{(4.44)^2} = .384 \]. With numerator d.f. = \( m - 1 = 10 - 1 = 9 \), and denominator d.f. = \( n - 1 = 5 - 1 = 4 \), we reject \( H_0 \) if \( f \geq F_{.05,9,4} = 6.00 \) or \( f \leq F_{.05,4,9} = \frac{1/3.63 = .275}. \text{Since } .384 \text{ is in neither rejection region, we do not reject } H_0 \text{ and conclude that there is no significant difference between the two standard deviations at level } \alpha = 0.10. \]