1. (a) $H_0 : \mu = 120$  $H_a : \mu \neq 120$  $\mu =$ OSU mean

(b) Yes, reject $H_0$ because C.I. with $\alpha = 0.16$ does not contain 120.

(c) $z_{0.05} = 1.64$. CI is $123.8 \pm \frac{(1.64)(10.3)}{5} = 123.8 \pm 2.96 = (120.84, 126.76)$

This interval is slightly shorter since $z_a < t_{a, 24}$, but for 24 df $t$ is fairly close to $z$.

(d) for $\frac{w}{2} = 2$, we will need more than 25 obs. This should be large enough to use $z$ in the formula. We use $s = 10.5$ as $\sigma$. If it is wrong, the length of the interval will not be exactly what we want, but as long as we use the observed $s$ for the interval, the confidence level will be correct.

2.

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFFF</td>
<td>5</td>
<td>4.7</td>
<td>0.6</td>
</tr>
<tr>
<td>ATC</td>
<td>5</td>
<td>6.9</td>
<td>0.8</td>
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$H_0 : \mu_2 - \mu_1 = 1$  $H_a : \mu_2 - \mu_1 > 1$

$T = \frac{(6.9 - 4.7) - 1}{\sqrt{\frac{(0.6)^2 + (0.8)^2}{5}}} = \frac{1.2}{1} = 2.68$

$T = \frac{1}{4} \left[ \frac{(0.6)^2 + (0.8)^2}{25} \right]^2 = 4 \left[ \frac{(0.6)^2 + (0.8)^2}{25} \right] = \frac{4(1)}{0.5392} = 7.42$  Use df = 7.  $t_{0.01, 7} = 2.998$

Do not reject $H_0$. Not strong evidence that $\mu_2 - \mu_1 > 1$.

3. (a) $n = 22$, $\alpha = 0.02$.

$\frac{(n - 1)S^2}{\chi^2_{a, n-1}}$, $\frac{(n - 1)S^2}{\chi^2_{1 - a, n-1}}$ = ($\frac{(21)(25.368)}{\chi^2_{0.01, 21}}$, $\frac{(21)(25.368)}{\chi^2_{0.99, 21}}$) = ($\frac{532.728}{38.93}$, $\frac{532.728}{8.897}$) = (13.68, 59.88)

(b) ($\sqrt{13.68}$, $\sqrt{59.88}$) = (3.70, 7.74)
4. (a) \( n = 37 \),
Explanatory is “return” for 1st 5 days of Jan. (x).
Response is full year’s “return” (y).
L.S. Equation: \( y = 7.932 + 3.440x \)

(b) (i) t-test: t-ratio for \( \beta_1 \) is 2.94, \( t_{0.025,35} = 2.03 \). Reject \( H_0 \) and conclude slope is not zero, i.e. early Jan. return can be used to predict full year return.

(ii) F test: \( F = \frac{1978.6}{228.1} = 8.67 > F_{1.35}(0.05) \approx 4.12 \). Reject.

(c) set \( x = 0 \), \( \hat{y}_0 = 7.932 = \hat{\beta}_0 \). S.D. of \( \hat{\beta}_0 \) is given as 2.500. 95% C.I. is \( 7.932 \pm (2.03)(2.5)\)
\[ 7.932 - 5.075 = (2.875, 13.007) \]

(d) Looking at (c), it would seem to be surprising. However, a prediction interval is wider than a C.I. and has as S. D. for \( x = 0 \),
\[ S_{pred} = \sqrt{MSE + (S.D.(\hat{\beta}_0))^2} = 15.31 \]
So the 95% Prediction Interval is: \( 7.932 \pm (2.03)(15.31) = 7.932 \pm 31.08 = (-23.14, 39.01) \)
From this viewpoint, a negative return would not be surprising.

(e) For \( x \neq 0 \), the formula for \( S(Prediction) \) is:
\[ S_{pred} = \sqrt{\left( MSE(1 + \frac{1}{n})\right) + \left( x - \bar{x} \right)^2 \left( S.E.(\hat{\beta}_1) \right)^2} \]
This gives: \( S_{pred} = \sqrt{\left( 228.1(1 + \frac{1}{37})\right) + \left( .5 - .246 \right)^2 \left( 1.168 \right)^2} = 15.31 \)
\[ \hat{y}_{0.5} = 7.932 + 3.440(0.5) = 9.652 \]. Also, \( t_{0.05,35} = 1.690 \).
Thus, the 90% P.I. is \( 9.652 \pm (1.690)(15.31) = 9.652 \pm 25.872 = (-16.220,35.524) \).

(f) ANOVA

<table>
<thead>
<tr>
<th>Source</th>
<th>d.f.</th>
<th>S.S.</th>
<th>M.S.</th>
<th>F</th>
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</thead>
<tbody>
<tr>
<td>Regression</td>
<td>1</td>
<td>1978.6</td>
<td>1978.6</td>
<td>8.67</td>
</tr>
<tr>
<td>Error</td>
<td>35</td>
<td>7983.5</td>
<td>228.1</td>
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</tr>
<tr>
<td>Total</td>
<td>36</td>
<td>9962.1</td>
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</tbody>
</table>

\( r^2 = \frac{SSR}{SST} = \frac{1978.6}{9962.1} = 0.199 \approx 20\% \). \( r = \sqrt{r^2} = \sqrt{0.199} = 0.446 \). Notice that the sign of \( r \) is positive because the sign of \( \hat{\beta}_1 \) is positive.

5. \( H_0 : \sigma_1^2 = \sigma_2^2 \) (or \( \frac{\sigma_1^2}{\sigma_2^2} = 1 \)) \hspace{1cm} H_a : \sigma_1^2 \neq \sigma_2^2 \)
\[ F = \frac{S_1^2}{S_2^2} \text{ or } \frac{S_2^2}{S_1^2} \quad \text{(whichever is >1)} \] \[ F = \frac{54}{48} = 1.27 = \frac{S_2^2}{S_1^2} \]. Since \( S_2^2 \) has 20 d.f. and \( S_1^2 \) has 15 d.f., compare F to \( F_{0.05,20,15} = 2.33 \) (note that \( F_{0.05,15,20} = 2.20 \), but is not the correct value for comparison). Since F<2.33, do not reject \( H_0 \).
6. a) Let \( \mu_1, \mu_2, \mu_3, \mu_4 \) be the mean lifetimes of the four brands.
\[ H_0: \mu_1=\mu_2=\mu_3=\mu_4 \quad H_a: \text{not all } \mu's \text{ are equal} \]

b) Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
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<tbody>
<tr>
<td>Brand</td>
<td>3</td>
<td>7.53\times10^4</td>
<td>2.51\times10^4</td>
<td>1.7075</td>
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<tr>
<td>Error</td>
<td>16</td>
<td>23.52\times10^4</td>
<td>1.47\times10^4</td>
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</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>31.05\times10^4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) Reject the hypothesis if \( F > F_{0.05, 3, 16} = 3.24 \). Since 1.71 < 3.24, we DO NOT reject \( H_0 \).