1. (20 points) To study copper deficiency in cattle, the copper values were determined both for cattle grazing in an area known to have excess concentrations of molybdenum and for cattle grazing in an area without any unusual concentrations. Using the data given below, carry out a test at significance level 0.10 to see whether the population standard deviations for the copper concentrations of the two groups are different. (You may assume normality of the populations.). Be sure to state your hypotheses.

<table>
<thead>
<tr>
<th>Area</th>
<th>Sample Size</th>
<th>Sample Mean</th>
<th>Sample SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Molybdenum</td>
<td>51</td>
<td>30.7</td>
<td>21.5</td>
</tr>
<tr>
<td>Nothing unusual</td>
<td>41</td>
<td>16.2</td>
<td>19.45</td>
</tr>
</tbody>
</table>

\[ H_0 : \sigma_1^2 = \sigma_2^2 \quad \text{or} \quad H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1 \quad \text{vs.} \quad H_a : \sigma_1^2 \neq \sigma_2^2 \quad \text{or} \quad H_a : \frac{\sigma_1^2}{\sigma_2^2} \neq 1. \]

The calculated test statistic is \[ f = \frac{S_1^2}{S_2^2} = \frac{(21.5)^2}{(19.45)^2} = 1.222. \]

numerator d.f. = \( m - 1 = 51 - 1 = 50 \),


denominator d.f. = \( n - 1 = 41 - 1 = 40 \)

, we reject \( H_0 \) if \( f \geq F_{0.05,50,40} = 1.66 \)

or \( f \leq F_{0.95,50,40} = \frac{1}{F_{0.05,40,50}} = \frac{1}{1.63} = 0.613 \).

Since 1.222 is in neither rejection region, i.e. \( 0.613 < f < 1.66 \),

we do not reject \( H_0 \)

and conclude that there is no significant difference between the two standard deviations at level \( \alpha = 0.10 \).
2. (25 points) A study includes the accompanying data on compression strength (lb) for a sample of aluminum cans filled with strawberry drink and another sample filled with cola. Calculate a 95% CI for the difference between true average compression strength for the strawberry drink cans and that for the cola cans. (You may assume that observations are normal and independent).

<table>
<thead>
<tr>
<th>Beverage</th>
<th>Sample Size</th>
<th>Sample Mean</th>
<th>Sample St. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strawberry drink</td>
<td>20</td>
<td>546</td>
<td>23</td>
</tr>
<tr>
<td>Cola</td>
<td>20</td>
<td>560</td>
<td>18</td>
</tr>
</tbody>
</table>

First, find the df:

\[ v = \frac{\left( \frac{S_1^2 + S_2^2}{m + n} \right)^2}{\frac{S_1^4}{m^2(m-1)} + \frac{S_2^4}{n^2(n-1)}} \]

and when \( m = n \), this simplifies to:

\[ v = \frac{(m-1)(S_1^2 + S_2^2)^2}{S_1^4 + S_2^4} = \frac{(19)(23^2 + 18^2)^2}{23^4 + 18^4} \]

\[ = \frac{19(529 + 324)^2}{279,841 + 104,976} = \frac{19(853)^2}{384,817} = \frac{19(727,609)}{384,817} = 19(1.891) = 35.925 \]

Use 35 df.

Need \( t_{0.025,35} = 2.030 \)

95% CI is

\[ (546 - 560) \pm 2.030 \sqrt{\frac{23^2 + 18^2}{20}} = -14 \pm 2.030 \sqrt{\frac{853}{20}} = -14 \pm 2.030 \sqrt{42.65} \]

\[ = -14 \pm 2.030 (6.53) = -14 \pm 13.26 \]

\[ = (-27.26, -0.74) \]
3. (20 points) Consider the accompanying data on breaking load (kg/25 mm width) for various fabrics in both an unabraded condition and an abraded condition. Use the paired $t$ test with $\alpha = .01$ to test $H_o : \mu_D = 0$ versus $H_a : \mu_D > 0$.

<table>
<thead>
<tr>
<th>Fabric</th>
<th>Sample Mean</th>
<th>Sample SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unabraded</td>
<td>25.6</td>
<td>48.8</td>
</tr>
<tr>
<td>Abraided</td>
<td>26.5</td>
<td>52.5</td>
</tr>
<tr>
<td>Difference</td>
<td>-0.9</td>
<td>-3.7</td>
</tr>
</tbody>
</table>

$d = 7.25, \quad s_D = 11.86$

Parameter of Interest: $\mu_D =$ true average difference of breaking load for fabric in unabraded or abraded condition.

$H_o : \mu_D = 0, \quad H_a : \mu_D > 0$

The value of the test statistic is: $t = \frac{\bar{d} - \mu_D}{s_D / \sqrt{n}} = \frac{7.25 - 0}{11.86 / \sqrt{8}} = 1.73$

The rejection region is: $t \geq t_{0.01,7} = 2.998$

Since $t$ is not $\geq 2.998$, we do not reject $H_o$.

The data do not indicate a difference in breaking load for the two fabric load conditions.
4. In their advertisements, an automobile manufacturer would like to claim that the gasoline mileage for its new hybrid car is greater than 55. In order to determine if this is a valid claim, a consumer organization selected 10 nonprofessional drivers to drive a car from Phoenix to Los Angeles. The average miles per gallon value for the 10 cars at the conclusion of the trip was \( \bar{x} = 55.81 \). Assume a normal distribution for measurements with known \( \sigma = 1.5 \).

(a) (10 points) State the question in terms of a hypothesis test and give the rejection region of an \( \alpha = .05 \) test. What is your conclusion?

Let \( \mu \) be true average mileage. \( H_0: \mu = 55, \quad H_a: \mu > 55 \)

Assume the normal distribution. \( n = 10, \quad \bar{x} = 55.81 \)

\[
Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{55.81 - 55}{1.5 / \sqrt{10}} = 0.81 \]

Reject if \( Z > z_{0.05} = 1.645 \).

Since 1.71 > 1.645, reject \( H_0 \). Conclude that true average mileage does exceed 55.

(b) (10 points) What is the probability of making a Type II error when \( \mu = 56 \)?

The rejection region is:

Reject if

\[
\bar{x} > 55 + (1.645)(0.474) = 55.780
\]

Thus, \( \beta(56) = P(Accept | \mu = 56) = P\left( \frac{\bar{x} - 56}{0.474} < \frac{55 - 56}{0.474} + \frac{(1.645)(0.474)}{0.474} \right) \)

\[
= P(Z < -2.108 + 1.645) = P(Z < -0.46) = 0.3228
\]

(c) (15 points) What sample size is needed so that the probability of making a Type II error when \( \mu = 56 \) will be 0.01?

For general \( n \), \( \beta(56) = P\left( \frac{\bar{x} - 56}{1.5 / \sqrt{n}} < \frac{\sqrt{n}(55 - 56)}{1.5} + 1.645 \right) \)

\[
= P(Z < -0.67 \sqrt{n} + 1.645) = P(Z < -2.326) = 0.01
\]

Hence,

\[
-0.67 \sqrt{n} + 1.645 = -2.326 \quad \text{or} \quad \sqrt{n} = \frac{2.326 + 1.645}{0.67} = 3.971 = 5.96
\]

or \( n = 35.48 \)

Use, \( n = 36 \).