SOLUTIONS FOR HOMEWORK 2

Chapter 2

*14a. \(P(A \cup B) = P(A) + P(B) - P(A \cap B)\)

So \(P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.8 + 0.7 - 0.9 = 0.6\)

b. \[
\begin{array}{c}
P(\text{shaded region}) = P(A \cup B) - P(A \cap B) = 0.9 - 0.6 = 0.3 \\
\text{Shaded region} = \text{event of interest} = (A \cap B') \cup (A' \cap B)
\end{array}
\]

*22.

a. \(P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.4 + 0.5 - 0.6 = 0.3\)

b. \(P(A_1 \cap A_2') = P(A_1) - P(A_1 \cap A_2) = 0.4 - 0.3 = 0.1\)

c. \(P(\text{exactly one}) = P(A_1 \cup A_2) - P(A_1 \cap A_2) = 0.6 - 0.3 = 0.3\)

23.
Assume that the computers are numbered 1 – 6 as described. Also assume that computers 1 and 2 are the laptops. Possible outcomes are (1,2) (1,3) (1,4) (1,5) (1,6) (2,3) (2,4) (2,5) (2,6) (3,4) (3,5) (3,6) (4,5) (4,6) and (5,6).

a. \(P(\text{both are laptops}) = P(\{(1,2)\}) = \frac{1}{15} = .067\)

b. \(P(\text{both are desktops}) = P(\{(3,4) (3,5) (3,6) (4,5) (4,6) (5,6)\}) = \frac{6}{15} = .40\)

c. \(P(\text{at least one desktop}) = 1 - P(\text{no desktops}) = 1 - P(\text{both are laptops}) = 1 - .067 = .933\)

d. \(P(\text{at least one of each type}) = 1 - P(\text{both are the same}) = 1 - P(\text{both laptops}) - P(\text{both desktops}) = 1 - .067 - .40 = .533\)

25.
\(P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.65\)

Similarly: \(P(A \cap C) = 0.55, P(B \cap C) = 0.60\)

\(P(A \cap B \cap C) = P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C)\)

= 0.98 - 0.7 - 0.8 + 0.65 + 0.55 + 0.6 = 0.53

Thus, \(P(A \cap B \cap C') = P(A \cap B) - P(A \cap B \cap C) = .65 - .53 = .12\), and

\(P(A \cap B' \cap C) = 0.02, \ P(A' \cap B \cap C) = 0.07\)
a. \( P(A \cup B \cup C) = 0.98 \) as given in the problem.
b. \( P(\text{none selected}) = 1 - P(A \cup B \cup C) = 1 - 0.98 = 0.02 \)
c. \( P(\text{only automatic transmission selected}) = 0.03 \) from the Venn diagram
d. \( P(\text{exactly one of the three}) = 0.03 + 0.08 + 0.13 = 0.24 \)

31a. \((\#\text{symphonies})(\#\text{concertos})=9(27)=243\)

b. \((\#\text{symphonies})(\#\text{concertos})(\#\text{quartets})=9(27)(15)=3645\). So such a policy could be carried out for 3645 successive nights, or approximately 10 years, without repeating exactly the same program.

*32a. \((\#\text{receivers})(\#\text{compact disc players})(\#\text{speakers})(\#\text{cassette deck})=5*4*3*4=240\)

b. \((\text{sony})(\text{sony})(\#\text{speakers})(\#\text{cassette deck})=1*1*3*4=12\)

c. \((\#\text{receivers-sony})(\#\text{compact disc players-sony})(\#\text{speakers})(\#\text{cassette deck-sony})=4*3*3*3=108\)

d. \# \text{with at least one sony} = (\text{total #}) - (\# \text{with no sony}) = 204 - 108 = 132\)

e. \( P(\text{at least one sony}) = \# \text{with at least one sony}/\text{total #} = 132/240 \)
\( P(\text{exactly one sony}) = (\# \text{with receiver as only sony} + \# \text{with cd player as only sony} + \# \text{with cassette deck as only sony} ) / (\text{total #}) = (1*3*3*4 + 4*1*3*3 + 4*3*3*1)/240 = 0.413 \)

33a. \( \binom{20}{5} = \frac{20!}{5!15!} = 15504 \)

b. \( \binom{8}{4} \binom{12}{1} = 840 \)

c. \( P(\text{exactly 4 have cracks}) = (\# \text{with 4 cracks}) / (\text{total #}) = 840/15504 = 0.0542 \)
d. \( P(\text{at least 4}) = P(\text{exactly 4}) + P(\text{exactly 5}) = 0.0542 + \frac{\binom{8}{12}}{\binom{5}{0}} = 0.0578 \)

\*38a. \( P(\text{selecting two 75 watt bulbs}) = \frac{\binom{6}{2} \binom{9}{1}}{\binom{15}{3}} = 0.2967 \)

b. \( P(\text{all three are the same}) = P(\text{all three are 40 watt}) + P(\text{all three are 60 watt}) + P(\text{all three are 75 watt}) = \)

\[ = \frac{\binom{4}{3} + \binom{5}{3} + \binom{6}{3}}{\binom{15}{3}} = 0.0747 \]

c. \( P(\text{one bulb of each type}) = \frac{\binom{4}{1} \binom{5}{1} \binom{6}{1}}{\binom{15}{3}} = 0.2637 \)

61. Before we can use Bayes’ Theorem we first need to find the following probabilities

\( P(0 \text{ defective in sample } | 0 \text{ defective in batch}) = 1 \)

\( P(1 \text{ defective in sample } | 0 \text{ defective in batch}) = 0 \)

\( P(2 \text{ defective in sample } | 0 \text{ defective in batch}) = 0 \)

\( P(0 \text{ defective in sample } | 1 \text{ defective in batch}) = \frac{\binom{9}{2}}{\binom{10}{2}} = 0.80 \)

\( P(1 \text{ defective in sample } | 1 \text{ defective in batch}) = \frac{\binom{9}{1}}{\binom{10}{2}} = 0.2 \)
P(2 defective in sample | 1 defective in batch) = 0

P(0 defective in sample | 2 defective in batch) = \frac{\binom{8}{2}}{\binom{10}{2}} = .622

P(1 defective in sample | 2 defective in batch) = \frac{\binom{2 \times 8}{1 \times 1}}{\binom{10}{2}} = .356

P(2 defective in sample | 2 defective in batch) = \frac{1}{\binom{10}{2}} = .022

To answer the questions use Bayes' Theorem and the above conditional probabilities

P(i defective in batch | j defective in sample) = \frac{P(j \text{ def. in sample} | i \text{ def. in batch})P(i \text{ def. in batch})}{\sum_{k=0}^{j} P(j \text{ def. in sample} | k \text{ def. in batch})P(k \text{ def. in batch})}

\textbf{a. P(0 defective in batch | 0 defective in sample)} = \frac{(1)(.5)}{(1)(.5) + (.8)(.3) + (.622)(.2)} = .578

P(1 defective in batch | 0 defective in sample) = \frac{(.8)(.3)}{(1)(.5) + (.8)(.3) + (.622)(.2)} = .287

P(2 defective in batch | 0 defective in sample) = \frac{(.622)(.2)}{(1)(.5) + (.8)(.3) + (.622)(.2)} = .144

\textbf{b. P(0 defective in batch | 1 defective in sample)} = \frac{(0)(.5)}{(0)(.5) + (.2)(.3) + (.356)(.2)} = 0

P(0 defective in batch | 1 defective in sample) = \frac{(.2)(.3)}{(0)(.5) + (.2)(.3) + (.356)(.2)} = .457

P(0 defective in batch | 1 defective in sample) = \frac{(.356)(.2)}{(0)(.5) + (.2)(.3) + (.356)(.2)} = .543
*102. Let B denote the event that a component needs rework. Then

$$P(B) = \sum_{i=1}^{3} P(B \mid A_i) \cdot P(A_i) = (.05)(.50) + (.08)(.30) + (.10)(.20) = .069$$

Thus, $P(A_1 \mid B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{P(B \mid A_1)P(A_1)}{P(B)} = \frac{(.05)(.50)}{.069} = .362$

$P(A_2 \mid B) = \frac{(.08)(.30)}{.069} = .348$

$P(A_3 \mid B) = \frac{(.10)(.20)}{.069} = .290$