Hw 6: Solutions to Required Problems

4.60. The mean number of nonword errors is
\[(0)(0.1) + (1)(0.2) + (2)(0.3) + (3)(0.3) + (4)(0.1) = 2.1,\]
and the mean number of word errors is
\[(0)(0.4) + (1)(0.3) + (2)(0.2) + (3)(0.1) = 1.\]

4.69. With \(R\) as the rod length and \(B_1\) and \(B_2\) the bearing lengths, we have
\[
\mu_{B_1+R+B_2} = 10 + 2 \times 2 = 14 \text{ cm and } \sigma_{B_1+R+B_2} = \sqrt{0.005^2 + 2 \times 0.001^2} = 0.005196 \text{ cm}.
\]

4.72. Independent: Weather conditions a year apart should be independent.
(b) Not independent: Weather patterns tend to persist for several days; today’s weather tells us something about tomorrow’s.
(c) Not independent: The two locations are very close together, and would likely have similar weather conditions.

4.76. (a) \(\mu_Y = \frac{1}{2} (\mu_{X_1} + \mu_{X_2}) = 100 \text{ m}, \) and \(\sigma_Y = \frac{1}{2} \sigma_{X_1+X_2} = \frac{1}{2} \sqrt{1.2^2 + 0.85^2} \approx 0.7353 \text{ m} \).
(b) \(\mu_W = \frac{1}{3} \mu_{X_1} + \frac{2}{3} \mu_{X_2} = 100 \text{ m}, \) and \(\sigma_W = \sqrt{\left(\frac{1}{3}\right)^2 \sigma_{X_1}^2 + \left(\frac{2}{3}\right)^2 \sigma_{X_2}^2} \approx 0.6936 \text{ m} \).

Calculus Supplement Exercise

4.8. \(E(X) = \int_{-3}^{3} x^3 \frac{\sqrt{18}}{90} dx = \frac{x^4}{72}\bigg|_{-3}^{3} = \frac{81}{72} - \frac{81}{72} = 0, \) which you could tell from the symmetry of \(f(x)\).

Thus, \(\sigma_X^2 = E[(X - \mu_X)^2] = E[X^2] - (E[X])^2 = E[X^2], \) and
\[
E[X^2] = \int_{-3}^{3} x^4 \frac{\sqrt{18}}{90} dx = \frac{x^5}{90}\bigg|_{-3}^{3} = \frac{243}{90} + \frac{243}{90} = \frac{27}{5} = 5.4.
\]

3.7 (a) In an observational study, we might take a sample and classify each subject as a wine drinker or a beer drinker. For an experiment, we would assign each subject to drink either wine or beer. In either case, we would then observe the health of the subjects over time. (b) Wine drinkers might be wealthier, better educated, have white-collar jobs, and have different dietary habits (“beer and pretzels” vs. “wine and cheese”).

3.30 The sketches requested in the problem are not shown here; random assignments will vary among students.

(a) Label the circles 1 to 6, then randomly select three (using Table B, or simply by rolling a die) to receive the extra CO$_2$. I used line 104 of Table B. The first 4 digits are: 5 2 7 1. We cannot use the 7 because it is more than 6. Therefore, we would treat areas 5, 2 and 1. Observe the growth in all six regions, and compare the mean growth within the three treated circles with the mean growth in the other three (control) circles.

(b) Select pairs of circles in each of three different areas of the forest (we are assuming that if the elements of each pair are close to each other they are of similar fertility). For each pair, randomly select one circle to receive the extra CO$_2$ (using Table B or by flipping a coin). For each pair, compute the difference in growth (treated minus control).

The mechanics of randomly choosing one of each pair could be:
. Label the two areas in each pair A and B. If the random number from Table B is even, then apply treatment to area A. Otherwise, apply the treatment to Area B. Alternatively, we could go along the table looking for either a 0 or a 1, ignoring the other digits. If we find a 0 before a 1, then treat area A. Otherwise, treat B.