

# Discussion Supersaturated Designs

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- 1 Screening
- 2 Problem of interactions
- 3 Two-stage group screening
- 4 Issues in use of supersaturated designs

Dennis and Denise

Thank you

for your interesting talks!

Supersaturated designs are used primarily for screening

- In screening, we try to identify a small number of “important”
  - factors
  - models
  - genes
  - molecules

out of a very large number of possibilities

# "Important"

"Important"  
in the context of factor screening  
means  
“stands out from the crowd”

It does not mean  
“has a non-zero effect”

This makes it possible to examine a large number of factors  
simultaneously.

# Supersaturated Designs

Good and Bad features:

- allow screening among a very large number of factors with relatively very few runs (e.g. Denise looked at 40 factors in 24 runs)
- usually need assume no large interactions and look only at main effects
- unless correlations between design columns are small, there may be some difficulty of untangling correlated effect estimates during screening

## FOLLOW-UP AND CONFIRMATION ARE ESSENTIAL!!

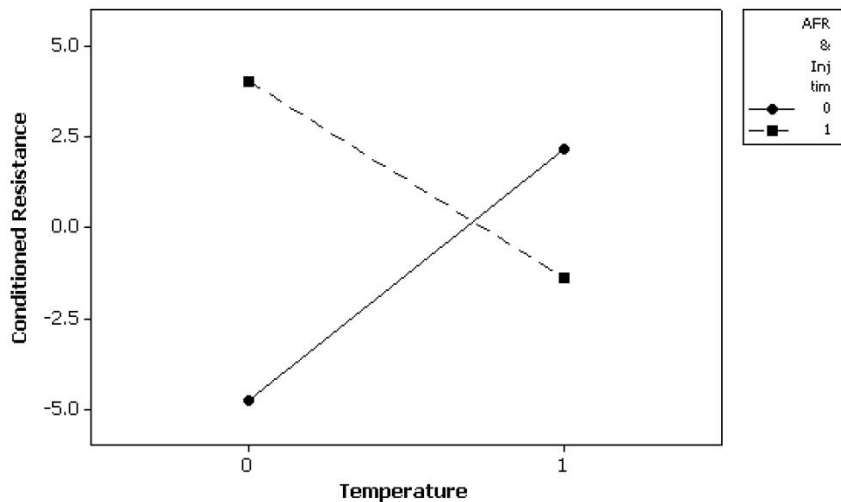
- follow-up runs needed for verification of importance of selected factors (i.e. check not selected by accident)
- follow-up experiment needed for model fitting (in my opinion)
- follow-up experiment needed for investigating interactions (between selected factors)

## Assumption of no interactions

- generally OK
  - if main effects are much larger than interactions
  - if interactions occur only between factors with large main effects
- but interactions can, and do, occur when there are no large main effects

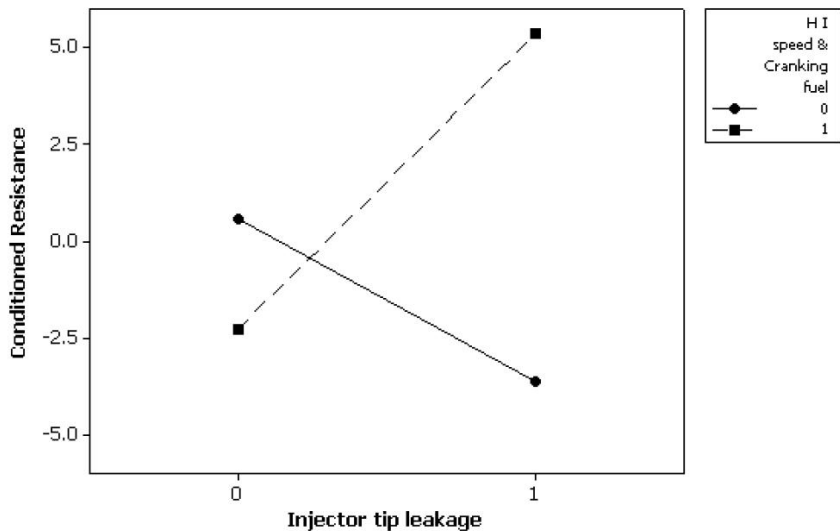
# Interactions with no large main effects

(a)



# Interactions with no large main effects

(b)



# Interactions, continued

- some supersaturated designs do allow estimation of interaction effects
- examples given by Wu (1993); Liu, Ruan and Dean (2006)
- but incorporation of interactions seems to be seldom advocated in the literature

# Two-Stage Group Screening

Two stage group screening provides a possible alternative to the use of supersaturated designs

- put factors into groups
- groups of main effects are aliased at stage 1 (Resolution II)
- groups of two-factor interactions are aliased at stage 1
- need to assume higher order interaction effects small
- at stage I, screen for important groups
- follow up with experiment on individual factors within the important groups (stage II)

# Two-Stage Group Screening, continued

- may require more observations than supersaturated designs
- easy to untangle aliasing provided very small number of important effects
- allows estimation of interactions

# Example of group screening

- Main effects model only. 40 factors in 24 runs
- Stage I
  - Put factors into 8 groups of size 5
  - Factors within a group all set at high level or all at low level, creating a “group factor” with two levels
  - Using the 8 group factors, run a Plackett-Burman design with 12 runs, leaving 3 d.f. for error
  - Analyze and select two, one or no important groups
- Stage II
  - Use the 10 or 5 factors in the important groups individually
  - Run a Plackett-Burman design with 12 runs, leaving 1 or 6 d.f. for error
  - Analyze
- Requires more observations to include interactions

# Possible errors

Possible errors in use of supersaturated designs include:

- 1 selection of non-active factors due to correlation of their design columns with those of active factors
- 2 non-selection of active factors due to fitting of correlated non-active factors
  - Bayesian model averaging and best subset selection avoid this problem
- 3 non-selection of active factors due to other correlated effects appearing more important

Similar errors 1 and 2 can occur in group screening

# Recommendations for Supersaturated Designs

- Number of observations should be at least twice the expected number of large effects
- correlations between design columns should be less than 0.33 if possible
  - Chen and Lin (1998)
  - Liu, Ruan and Dean (2006)
- non-linear main effects can still be screened at two levels
- design optimality may be less important than a good method of analysis

# Supersaturated and Group Screening

- Which is the better screening method for main effects?
- Which is the best analysis method?
- Which is the best design for the chosen analysis method?

Only know partial answers to these questions

# Best Screening Method?

Which is best?

Supersaturated or Group Screening?

- not known at present
- for even larger number of factors, may need to combine the methods
- i.e. two-stage group screening using supersaturated designs at each stage

# Best Analysis Method?

- Non-Bayesian

- all subset selection usually too large to be feasible
- forward selection (or stepwise) repeated many times with randomly ordered design columns and model averaging
- methods based on contrasts
  - Holcomb, Montgomery, Carlyle (2003)
- penalized least squares
  - Li and Lin (2002)

- Bayesian

- SSVS and model averaging
  - c.f. Chipman, Hamada, Wu, 1997
  - c.f. Meyer and Wilkinson, 1998
- SSVS/IBF
  - Beattie, Fong, Lin (2002)

# Best Design Construction Criteria?

- design criteria should match the goal of the experiment
  - criteria such as maximize probability of identifying active factors
    - Beattie, Fong and Lin (2002)
    - Allen and Brnshteyn (2003)
    - Vine, Lewis and Dean (2005)
  - are probably the most sensible
- criteria such as  $Es^2$  optimality,  $D$ -optimality etc. are surrogates for the true criteria of interest
  - trying to achieve small column correlations

# Some Useful References

## Overview and comparisons

- Survey: Gilmour (2006) In “Screening:....”, Springer
- Analysis: Li and Lin (2003) J. Data Sci.
- Design: Liu and Dean (2004) Technometrics

## Analysis aspects

- Chipman, Hamada and Wu (1997) Technometrics
- Meyer and Wilkinson (1998) Commun. Stat. - Th. & Meth.
- Li and Lin (2002) Stat.Prob. Ltr.
- Beattie, Fong and Lin (2002) Technometrics
- Holcomb, Montgomery and Carlyle (2003) J. Quality Techn.

## Design issues

- Chen and Lin (1998) JSPI
- Allen and Bernshteyn (2003) Technometrics
- Ruan, Liu and Dean (2006) In revision

## Two-Stage Group Screening

- Vine, Lewis and Dean (2005) *Statistica Sinica*
- Dupplaw et al. (2004)  
J. Computing & Inf. Sci. in Engineering