Discussion
Supersaturated Designs

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1 Screening
2 Problem of interactions
3 Two-stage group screening
4 Issues in use of supersaturated designs
Dennis and Denise

Thank you

for your interesting talks!
Supersaturated designs are used primarily for screening.

In screening, we try to identify a small number of “important”
- factors
- models
- genes
- molecules

out of a very large number of possibilities.
"Important" in the context of factor screening means “stands out from the crowd”

It does not mean “has a non-zero effect”

This makes it possible to examine a large number of factors simultaneously.
Good and Bad features:

- allow screening among a very large number of factors with relatively very few runs (e.g. Denise looked at 40 factors in 24 runs)

- usually need assume no large interactions and look only at main effects

- unless correlations between design columns are small, there may be some difficulty of untangling correlated effect estimates during screening
FOLLOW-UP AND CONFIRMATION ARE ESSENTIAL!!

- follow-up runs needed for verification of importance of selected factors (i.e. check not selected by accident)
- follow-up experiment needed for model fitting (in my opinion)
- follow-up experiment needed for investigating interactions (between selected factors)
Assumption of no interactions

- generally OK
  - if main effects are much larger than interactions
  - if interactions occur only between factors with large main effects

- but interactions can, and do, occur when there are no large main effects
Interactions with no large main effects
Interactions with no large main effects
some supersaturated designs do allow estimation of interaction effects

effects given by Wu (1993); Liu, Ruan and Dean (2006)

but incorporation of interactions seems to be seldom advocated in the literature
Two stage group screening provides a possible alternative to the use of supersaturated designs.

- put factors into groups
- groups of main effects are aliased at stage 1 (Resolution II)
- groups of two-factor interactions are aliased at stage 1
- need to assume higher order interaction effects small
- at stage I, screen for important groups
- follow up with experiment on individual factors within the important groups (stage II)
Two-Stage Group Screening, continued

- may require more observations than supersaturated designs
- easy to untangle aliasing provided very small number of important effects
- allows estimation of interactions
Example of group screening

- Main effects model only. 40 factors in 24 runs

- Stage I
  - Put factors into 8 groups of size 5
  - Factors within a group all set at high level or all at low level, creating a “group factor” with two levels
  - Using the 8 group factors, run a Plackett-Burman design with 12 runs, leaving 3 d.f. for error
  - Analyze and select two, one or no important groups

- Stage II
  - Use the 10 or 5 factors in the important groups individually
  - Run a Plackett-Burman design with 12 runs, leaving 1 or 6 d.f. for error
  - Analyze

- Requires more observations to include interactions
Possible errors

Possible errors in use of supersaturated designs include:

1. selection of non-active factors due to correlation of their design columns with those of active factors

2. non-selection of active factors due to fitting of correlated non-active factors
   - Bayesian model averaging and best subset selection avoid this problem

3. non-selection of active factors due to other correlated effects appearing more important

Similar errors 1 and 2 can occur in group screening
Recommendations for Supersaturated Designs

- Number of observations should be at least twice the expected number of large effects.

- Correlations between design columns should be less than 0.33 if possible.
  - Chen and Lin (1998)
  - Liu, Ruan and Dean (2006)

- Non-linear main effects can still be screened at two levels.

- Design optimality may be less important than a good method of analysis.
Which is the better screening method for main effects?

Which is the best analysis method?

Which is the best design for the chosen analysis method?

Only know partial answers to these questions
Which is best?

Supersaturated or Group Screening?

- not known at present

- for even larger number of factors, may need to combine the methods

- i.e. two-stage group screening using supersaturated designs at each stage
Best Analysis Method?

- Non-Bayesian
  - all subset selection usually too large to be feasible
  - forward selection (or stepwise) repeated many times with randomly ordered design columns and model averaging
  - methods based on contrasts
  - penalized least squares
    - Li and Lin (2002)

- Bayesian
  - SSVS and model averaging
  - SSVS/IBF
    - Beattie, Fong, Lin (2002)
Best Design Construction Criteria?

- Design criteria should match the goal of the experiment
  - Criteria such as
    - Maximize probability of identifying active factors
      - Beattie, Fong and Lin (2002)
      - Vine, Lewis and Dean (2005)
    - Are probably the most sensible
  - Criteria such as $E_{s^2}$ optimality, $D$-optimality etc.
    - Are surrogates for the true criteria of interest
      - Trying to achieve small column correlations
Some Useful References

Overview and comparisons

Analysis aspects
- Chipman, Hamada and Wu (1997) Technometrics
- Beattie, Fong and Lin (2002) Technometrics

Design issues
- Chen and Lin (1998) JSPI
- Allen and Bernshteyn (2003) Technometrics
- Ruan, Liu and Dean (2006) In revision
Two-Stage Group Screening

- Vine, Lewis and Dean (2005) Statistica Sinica
- Dupplaw et al. (2004)
  J. Computing & Inf. Sci. in Engineering