

Estimating Heterogeneous EBA and Economic Screening Rule Choice Models

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Abstract

Consumer choice in surveys and in the marketplace reflects a complex process of screening and evaluating choice alternatives. Behavioral and economic models of choice processes are difficult to estimate when using stated and revealed preferences because the underlying process is latent. This paper introduces Bayesian methods for estimating two behavioral models that eliminate alternatives using specific attribute-levels. The elimination by aspects theory postulates a sequential elimination of alternatives by attribute levels until a single one, the chosen alternative remains. In the economic screening rule model, respondents screen out alternatives with certain attribute levels, and then choose from the remaining alternatives using a compensatory function of all the attributes. The economic screening rule model gives an economic justification as to why certain attributes are used to screen alternatives. A commercial conjoint study is used to illustrate the methods and assess their performance. In this dataset, the economic screening rule model outperforms the EBA and other standard choice models.

Keywords: elimination by aspects; consideration sets; attribute screening; noncompensatory decision processes; conjoint analysis; hierarchical Bayes

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1. Introduction

Marketing researchers have drawn on multiple disciplines to describe consumer choice. Persistent themes from behavioral decision research are that respondents are different, or heterogeneous, and are "cognitive misers" who adopt decision heuristics to cope with complex choice tasks. This view is consistent with economic theory where the cost of thinking is included as an element of the budget constraint. Estimating these models, however, has proven difficult particularly when a decision is described in terms of a sequence of events that are not observed by the researcher or by constraints or thresholds that are latent. The purpose of this paper is to propose and demonstrate methods for estimating two models of choice motivated by behavioral decision theory – the elimination by aspects (EBA) model proposed by Tversky (1972) and an economic screening rule model where the screening rule is the result of a trade-off between cognitive effort and expected utility.

The EBA model assumes that consumers eliminate choice alternatives that do not have desired attributes or aspects. The first aspect, or product feature, is selected with a probability proportional to its relative importance, all alternatives without that aspect are eliminated, and the process is repeated until only a single alternative remains. The EBA model has intuitive appeal because the amount of cognitive processing associated with attribute based choice is considered to be less than with compensatory choice models where consumers consider all the alternatives on all the attributes. However, the choice alternatives in an EBA model can be selected through several different orderings of aspects used to eliminate the alternatives. The EBA has a closed form, but a complicated choice probability, with the complexity increasing as the number of alternatives and aspects used to describe alternatives increases. Published studies of EBA

models usually include a small number of attributes and/or alternatives and have not dealt with the issue of respondent heterogeneity.

Economic screening rule models assume respondents screen out alternatives with relatively low levels of utility, and then engage in effortful processing of the remaining alternatives. Gilbride and Allenby (2004) propose methods for estimating choice models with conjunctive, disjunctive and compensatory screening rules, where the first two rules involve thresholds for attribute-levels, and the latter rule is based on the overall utility of an offering being greater than a threshold value. The conjunctive model, which requires that an alternative be acceptable on all relevant attributes in order to be included in the final choice set, was the best fitting model in their study. This paper extends the conjunctive screening rule model by providing an economic rationale for the attribute-level thresholds. Screening rules can result in a loss of utility when an alternative is eliminated on the basis of a single attribute, while consideration of all attributes might reveal it as the most preferred alternative. The economic screening rule model maximizes the number of screening attributes subject to a constraint on the expected loss in utility.

The purpose of this paper is three-fold. First, a simulation based methodology is introduced to estimate the EBA model. This method overcomes the difficulties in specifying the choice probability that have limited applications of the EBA model and its empirical comparison to other choice models. The simulation is facilitated by using a new method for generating quasi-random numbers, the Modified Latin Hypercube Sampling (MLHS) plan proposed by Hess, Train, and Polak (2005). The MLHS is explained and compared to standard approaches in the context of estimating EBA choice probabilities.

Second, an economic screening rule model is proposed that provides a cost/benefit rationale to attribute based screening rules. This model results in fewer parameters than comparable screening rule models. Heterogeneity is introduced into both models via a hierarchical Bayes structure and Markov-chain Monte Carlo (MCMC) methods are detailed in the appendix for estimating the models.

Third, the results of an empirical application using a choice based conjoint study are presented. The performance of the EBA, economic screening rule, conjunctive screening rule, and the standard economic choice model (as represented by the multinomial logit model) are compared. In this data set, the economic screening rule model provided the best fit to the data. The empirical application demonstrates that the proposed models can be estimated using data commonly collected in marketing research studies.

The paper proceeds as follows. In the next section, the EBA model is reviewed and the simulation based estimation methodology is detailed. The following section develops the economic screening rule model and derives the likelihood function using assumptions consistent with standard discrete choice models. The fourth section provides a brief comparison of the EBA and economic screening rule model. This section presents results from simulated datasets showing that the models can identify the correct data generating process. The fifth section presents results from the empirical application and the paper closes with a discussion and suggested extensions of this research.

2. Estimating the EBA Model

The elimination by aspects (EBA) model proposed by Tversky (1972) is one of a number of choice models offered as counter-examples to the rational choice theory of economics. This

stream of literature begun by Simon (1955) focuses on how consumers actually make choices given their limited ability to accumulate, process, and make optimal decisions using all the information available in the market place. Bettman, Luce, and Payne (1998) provide an overview of the information processing approach to studying consumer choices. In this section we propose a method to estimate the EBA model with stated preference data.

The EBA model characterizes consumer choice as a covert elimination process. Choice alternatives are described by the attributes, or aspects that they possess. A consumer places different levels of importance, or weight, on each of these aspects. The choice process begins with the consumer probabilistically selecting one of the aspects; the probability of selecting a given aspect is proportional to its weight. All alternatives without that aspect are eliminated from the choice set; if only one alternative remains, it is chosen. If more than one alternative remains, a second aspect is chosen, again with probability proportional to its weight relative to the weight of the remaining aspects. The elimination process continues until only one alternative remains.

The following example is adapted from Maddala (1983, p. 65) and is used to illustrate the process and the choice probability. Formal mathematical treatments can be found in Tversky (1972) and Batsell, et al. (2003). Suppose there are three alternatives described on 7 aspects that are shared by the alternatives:

$$\begin{aligned} A_1 &= \{\alpha_1, \alpha_{12}, \alpha_{13}, \alpha_{123}\} \\ A_2 &= \{\alpha_2, \alpha_{12}, \alpha_{32}, \alpha_{123}\} \\ A_3 &= \{\alpha_3, \alpha_{13}, \alpha_{32}, \alpha_{123}\} \end{aligned}$$

where α_i denotes the weight, or importance, of the aspect to be estimated. Aspect α_1 is unique to alternative A_1 , aspect α_{12} is shared by A_1 and A_2 , and so on. The use of aspect α_{123} in the EBA

model would not eliminate any alternative, it can be ignored. Alternative A_1 can be selected via a number of possible scenarios: α_1 could be selected as the first aspect and all other alternatives eliminated; α_{13} could be selected, eliminating A_2 and then either α_1 or α_{12} could be selected eliminating A_3 ; or, α_{12} could be selected, eliminating A_3 and then either α_1 or α_{13} could be selected eliminating A_2 . Let S equal the sum over the six relevant α 's. Then

$$P(A_1) = \frac{\alpha_1}{S} + \left(\frac{\alpha_{13}}{S} \right) \left(\frac{\alpha_1 + \alpha_{12}}{\alpha_1 + \alpha_{12} + \alpha_2 + \alpha_{23}} \right) + \left(\frac{\alpha_{12}}{S} \right) \left(\frac{\alpha_1 + \alpha_{13}}{\alpha_1 + \alpha_{13} + \alpha_3 + \alpha_{23}} \right) \quad (1)$$

As the number of alternatives and aspects increases, the number of parameters and the complexity of specifying (1) for each individual for each choice occasion increases. As a result, there have been few published studies in marketing which have estimated the EBA model.

Another issue for marketing researchers is how to specify the aspects. In the above example, α_1 , α_2 , and α_3 are alternative specific aspects and α_{ij} are aspects shared by alternative i and j . Any collections of choice alternatives can be assigned "general" aspects in this fashion. Batsell et al. (2003) estimate the EBA model using repeated observations from a single subject using a "general" aspects coding for 5 brands of snack foods. They use observed choice probabilities and show that the differences in these choice probabilities can be linearly related to the parameters in the EBA model. This approach requires the respondent to evaluate all possible choice sets repeatedly in order to get estimates of the choice probabilities. Interpretation of the shared aspects (e.g. α_{12}) is left to the analyst. Manrai and Sinha (1989) provide an application where perceptual data is collected from respondents and two factor scores for each alternative are used to represent the aspects.

Marketing researchers, however, are frequently interested in the influence of particular attributes such as price, color, size, etc. as opposed to "general" aspects. Fader and McAlister (1990) estimate individual level EBA models where the attributes are limited to brand and promotional status. In their parameterization each brand has a specific aspect, brand, and a common attribute, promotional status. Consumers choose either from the set of all acceptable brands or from the subset of brands on promotion. Using either "general" aspects or limiting the number of attributes greatly simplifies evaluation of equation (1).

In the next section a Bayesian method of estimating a heterogeneous EBA model is proposed. This method uses a simulation based method for evaluating the likelihood and does not require the construction and evaluation of equation (1). It therefore is appropriate for marketing studies such as discrete choice conjoint where there are several choice alternatives described on multiple attributes each with different levels.

Model Estimation

The model is estimated by replicating the EBA selection process repeatedly for a given set of parameters and calculating the choice probability. Let k index the set of attributes and j enumerate the levels within the attribute. So if the attributes are Brand, Size, and Color, $K = 3$; the corresponding attribute levels may be {Brand A, Brand B, Brand C}, {Large, Small}, {White, Black} and J_k ranges from 2 to 3. Let α_{kj} be the weight given to attribute k level j . These weights are the parameters of interest and they correspond to distinct attribute levels and are not the "general" aspects defined above. For a given choice occasion, the choice process begins by selecting the first attribute level, denoted as $\{k_j\}^1$:

$$\Pr(\{kj\}^1) = \frac{\alpha_{kj}}{\sum_{k=1}^K \sum_{j=1}^{J_k} \alpha_{kj}} \quad (2)$$

All alternatives that do not have attribute level $\{kj\}^1$ are eliminated. All remaining alternatives have the same level of attribute k^1 and therefore attribute k^1 no longer influences the selection process. The second attribute level $\{kj\}^2$ is selected with probability:

$$\Pr(\{kj\}^2) = \frac{\alpha_{kj}}{\sum_{\substack{k=1 \\ k \neq k^1}}^K \sum_{j=1}^{J_k} \alpha_{kj}} \quad (3)$$

The process is repeated until a single alternative remains, and that is designated the selected alternative, y_i^* . The maximum number of steps in the selection process is K , the number of attributes. The algorithm is summarized as:

For $t = 1$ to K :

- i) select attribute level $\{kj\}^t$ with probability $\frac{\alpha_{kj}}{\sum_{\substack{k=1 \\ k \notin \{k^{t-1}, k^{t-2}, \dots\}}}^K \sum_{j=1}^{J_k} \alpha_{kj}}$
- ii) Eliminate all alternatives that do not have attribute level $\{kj\}^t$
- iii) If only one alternative, i , remains set $y_i^* = 1$; otherwise increase t and go to i)

where the notation $\{k^{t-1}, k^{t-2}, \dots\}$ represents the set of previously selected attributes.

The choice probability is given by repeating the algorithm i) to iii) many times and calculating the selection frequency. Let $y_{im} = 1$ if alternative i was chosen in choice set m by the respondent. Let $s = 1$ to S where S is the total number of times i) to iii) is repeated and $y_{ims}^* = 1$

indicate that alternative i was selected in simulation s . The indicator function $I(y_{im} = y_{ims}^*) = 1$ if the chosen alternative matches the alternative selected by the algorithm on simulation s , and 0 otherwise. Then:

$$l_m = \Pr(y_{im} = 1) = \frac{\sum_{s=1}^S I(y_{im} = y_{ims}^*)}{S} \quad (4)$$

and the conditional likelihood over M choice occasions is given by $l = \prod_{m=1}^M l_m$. With the specification of the conditional likelihood, standard MCMC methods can be used to obtain samples from the posterior distributions of the parameters. Appendix 1 discusses methods of improved estimation of equation (4) using quasi-random number sequences.

The simulation method outlined in i) to iii) and (4) does not require the analyst to specify all the possible paths that could lead to $y_{im} = 1$. Repeating the simulation many times by randomly selecting $\{k_j\}^1, \{k_j\}^2, \dots, \{k_j\}^K$ effectively integrates over the space of allowable choice paths. Eliminating the need to specify (1) for each respondent in each choice occasion simplifies estimating the EBA model in studies such as conjoint research which involve numerous respondents, alternatives, attributes, and attribute levels. In the above algorithm, if an attribute level that is common to all alternatives is selected as $\{k_j\}^t$, then no alternative is eliminated and the algorithm moves on to the next attribute. In some instances, an attribute level may be selected as $\{k_j\}^t$ which is not present in any of the remaining alternatives. If "None" is part of the choice set, then it is selected in this instance. If "None" is not part of the choice set,

then any attribute levels $\{kj\}$ not present in any of the alternatives are excluded from the selection probability in step i).

A Bayesian approach to modeling heterogeneity is used as compared to previous studies that estimated individual level fixed effect models. Rossi and Allenby (2003) provide a discussion of the use of Bayesian methods to model heterogeneity. In this application, heterogeneity is introduced by subscripting the importance weights with h signifying a particular household, α_{hkj} . The weights must be greater than 0 for the selection probability in i) to make sense so we specify $\alpha_h = \exp(\alpha_h^*)$ and model heterogeneity as $\alpha_h^* \sim N(\overline{\alpha^*}, V_{\alpha^*})$. Multiplying the weights by a common constant leaves the selection probability in i) unchanged so the model is unidentified. This is not a problem for Bayesian estimation methods as long as a function of the parameters is identified. In this case $\Pr(\{kj\}^1)$ is identified and the posterior distribution of $\Pr(\{kj\}^1)$ is reported for each $\{kj\}$. This effectively imposes the identifying restriction $\sum \alpha_{hjk} = 1$ without complicating the MCMC sampler. Post-processing the draws of the sampler and reporting the identified function of the parameters was suggested by Edwards and Allenby (2003).

The EBA choice model is premised on a rather minimalist behavioral theory: sequentially eliminate alternatives that do not have the aspects or attributes that you want. This basic EBA theory was expanded by Tversky and Sattath (1979) and Gensch and Ghose (1992) to allow aspects to form natural groupings or hierarchies; a consumer then chooses between higher level hierarchies moving down to lower levels of the hierarchy or the "tree" until one alternative remains. A slightly more involved behavioral theory is that a consumer first screens out unacceptable alternatives based on particular attributes and then chooses among the remaining alternatives using a more effortful, compensatory second stage. Gilbride and Allenby (2004)

review two stage decision processes and provide statistical methods to estimate various screening rule models. In the next section, an economic model is used to determine why certain attributes are used to screen alternatives and how the choice is made from the remaining alternatives.

3. Estimating Economic Screening Rule Models

The economic screening rule model is based on the premise that the presence of an attribute-level may indicate that an offering is undesirable and not worth evaluating. Let ψ_i denote the indirect utility for offering i which is identified by the vector of its attribute levels x_i . We assume this vector includes an indicator for log price as an element. The mapping between ψ_i and x_i is represented by $\psi_i(x_i)$. The set of product offerings can be collected into a matrix $X = (x_1, x_2, \dots, x_n)$ where each column corresponds to a different offering, $1, \dots, n$. $\psi(X)$ represents the vector where the n^{th} element corresponds to the indirect utility of the n^{th} product offering. We assume that it takes cognitive effort on the part of the decision maker to interpret, encode, and map the matrix X to the vector of indirect utility ψ on any given choice occasion. This "cost of thinking approach" is discussed in the marketing literature by Hauser and Wernerfelt (1990), Roberts and Lattin (1991), Mehta et al. (2003), and Shugan (1980).

Decision makers can reduce the amount of cognitive effort by limiting the amount of information in X that is evaluated. The presence of any unacceptable element in x_i eliminates the need to further process the other elements in x_i or map it to ψ_i . We therefore assume that a consumer minimizes his/her cognitive effort by maximizing the number of screening attributes. Attribute-level " j " is used to screen out, or delete alternatives if:

$$d_j = 1 \text{ if } E_x[\text{Max}(\psi(X)) - \text{Max}(\psi(X_{-j}))] < \gamma \quad (5)$$

where $\psi(X)$ is a vector with n^{th} element equal to the indirect utility of choice alternative n , and $\psi(X_{-j})$ is a vector of indirect utilities that excludes all offerings with attribute-level j . γ is the amount of indirect utility a decision maker is willing to forgo in order to simplify the choice task. γ is inversely related to the amount of cognitive effort the decision maker is willing to spend: as γ approaches 0, no alternatives are screened out and the full matrix X is evaluated. The expectation in equation (5) is with respect to the respondent's beliefs about the distribution of attribute levels in the market, $\pi(x)$. An attribute-level is used to screen alternatives if the expectation of the maximum utility of a reduced choice set, excluding offerings with the attribute-level, is within γ of the full choice set. The choice model is then:

$$\text{Choose alternative } i \text{ if } \psi_i(x_i) > \psi_n(x_n) \text{ for all } n \text{ such that } d'x_n = 0 \quad (6)$$

where d is a vector with elements d_j corresponding to the screening criteria in equation (5). The screening rule requires that considered alternatives contain no unacceptable attribute-levels, or equivalently, that considered alternatives are comprised of only acceptable attribute-levels.

The model assumes a consumer utilizes his/her beliefs about the joint distribution of attribute-levels, $\pi(x)$, to determine which attributes are unacceptable and therefore which alternatives to screen out of any given choice set. The model is consistent with a two stage decision problem where the screening rule is the solution to a previous cost-benefit analysis. It is similar to the conjunctive screening rule model of Gilbride and Allenby (2004) but stipulates the model that leads to an attribute being either acceptable or unacceptable. As will be seen below in the empirical application, this results in a significant reduction in the number of parameters to be

estimated. Unlike other consideration set models that use a compensatory model across attributes to determine whether or not an alternative is included in the choice set, the proposed model results in a decision rule based on the levels of individual attributes.

Anecdotal evidence and the results of Gilbride and Allenby (2004) suggests that consumers may only focus on one or two attributes, such as brand or price, and adopt a screening rule based only on those attributes. The screening rule model implies that this is a reasonable strategy. For instance, if a particular brand is systematically associated with lower levels of other desirable attributes, then the loss of expected utility from excluding that brand from the choice set may be lower than the cost of evaluating all the attributes associated with the brand. Similarly, the psychological benefits of consuming a particular brand may be so low that even at the lowest price and highest levels of all other specified attributes, a consumer may never choose that brand. In either of these situations it makes sense for the consumer to adopt a decision rule that focuses on a particular attribute (e.g brand) and exclude alternatives with that attribute from the final choice set.

Model Estimation

Marketing researchers observe the actual choices of a consumer and a potentially incomplete listing of the product attributes. We parameterize the indirect utility function via $V_i = \beta'x_i + \varepsilon_i$ and use the error term ε to account for this uncertainty. Equation (5) is now given by:

$$d_j = 1 \text{ if } E_x[E_\varepsilon[\text{Max}(\beta'X+\varepsilon) - \text{Max}(\beta'X_{-j}+\varepsilon)]] < \gamma \quad (7)$$

where ε is a vector of error terms that we assume to be distributed iid standard extreme value.

The inner expectation is given by the expression (see Anderson, de Palma, and Thisse, 1992):

$$E_\varepsilon[\text{Max}(\beta'X + \varepsilon)] = \ln(\mathbf{1}' \exp[\beta'X]) + \delta \quad (8)$$

where $\mathbf{1}$ is vector of one's and δ represents Euler's constant, leading to the screening rule:

$$d_j = 1 \text{ if } E_x[\ln(\mathbf{1}' \exp[\beta'X]) - \ln(\mathbf{1}' \exp[\beta'X_{\cdot j}])] < \gamma \quad (9)$$

Evaluating equation (9) requires integrating over the beliefs about the distribution of attribute-levels. The empirical study in this paper relies on the pragmatic solution of using the empirical distribution of choice sets faced by the consumer in the study. Thus, the evaluation of equation (9) proceeds by averaging the expression in brackets on the left side of the inequality over the choice sets in the study. The choice probability is then:

$$\Pr(y_i = 1) = \frac{\exp[\beta' x_i]}{\sum_{\substack{n=1 \\ d'x_n=0}}^N \exp[\beta' x_n]} \quad (10)$$

Heterogeneity is introduced with a random-effect model for the β and γ parameters: $\beta_h \sim N(\bar{\beta}, V_\beta)$, and, since γ_h must be greater than 0, we specify $\gamma_h = \exp(\gamma_h^*)$ and let $\gamma_h^* \sim N(\bar{\gamma}^*, \sigma_{\gamma^*}^2)$.

We use diffuse but proper priors for the hyper-parameters. MCMC methods are used to obtain

samples from the posterior distributions of all parameters and details are contained in the appendix.

4. Distinguishing Features of EBA and the Economic Screening Rule Models

The EBA and economic screening rule model involve attribute based decision processes but the underlying nature of the decision models is different. In the EBA model, individual attributes are used to eliminate alternatives until only a single one remains. However, as illustrated in step i) of the algorithm, the attribute levels that are used will change from choice occasion to choice occasion as well as the order in which they are used to eliminate alternatives. Both the attribute levels that are used and the ordering can result in different alternatives being selected from identical choice sets; consequently, the EBA model is inherently probabilistic.

In the economic screening rule model, certain attribute levels are used to eliminate alternatives from the choice set, but the remaining alternative are evaluated using all the attribute levels. The screening attributes do not change from choice set to choice set and the order in which the screening takes place does not matter. Given the consumer's current beliefs about the distribution of choice sets, alternatives, and attribute levels, the economic screening rule is a deterministic choice model. It is only the absence of the researcher's knowledge of all possible attributes and environmental factors that lead to probabilistic models for economic choice models in general and the economic screening rule model in particular. Anderson et al. (1992), Tversky (1972), and Tversky and Sattath (1979) provide discussion of the distinction between stochastic and non-stochastic choice models.

A growing literature suggests that decision makers may adopt different decision strategies depending on the choice context. Differences in the number of alternatives, number of

attributes, the need or desire to be more accurate, etc. may all influence the particular strategy used; see Payne, Bettman, and Johnson (1992) and Bettman et al. (1998) for theories, experimental evidence, and research propositions relating to these issues. Differences in perspective may also lead researchers to specify stochastic versus non-stochastic choice models. Given that only stated and revealed preferences are available for analysis, it is important to establish the ability to identify each of these models. Moreover, the data should be representative of that which is encountered in marketing settings – i.e., relatively short data histories coupled with heterogeneous respondents.

Table 1 reports on a simulation study used to demonstrate the ability to identify the models. Three simulated data sets were generated consisting of 300 heterogeneous respondents, facing 10 choice tasks, each with 6 alternatives. The alternatives were described on 4 attributes: one with three levels and the remaining attributes at two levels. The choices for the first data set were generated according to the EBA decision model; choices for the second data set were generated according to a multinomial logit (MNL) or the standard economic choice model with extreme value error terms; and choices for the third data set were generated according to the economic screening rule model described in Section 3. Each dataset was used to estimate each of the models, with the true model listed on the vertical dimension of the table, and the estimated model along the horizontal dimension. The log marginal density for each model/dataset combination is reported in each cell.

===== Table 1 =====

The diagonal elements in table 1 represent the correct model applied to the appropriate data set, and indicate that the models can be identified by the data. The EBA model produces the largest log marginal density (LMD) for the EBA data (-4,150.3) compared to the MNL (-4,219.4)

and the economic screening rule model (-4,235.4). Similar results apply for the other data sets and models. One anomaly occurs when the screening rule model is used with the MNL data. The LMD for the screening rule model is (-2,957.5) compared to (-2,964.5) for the MNL model with the MNL data. However, note that in the screening rule model, if γ_h is set sufficiently low, all attributes pass the screening rule and no alternatives are screened out. This is the empirical result when the screening rule model was used with the MNL data: the posterior distribution of the parameters implied that virtually no alternatives were screened out of any choice sets (less than .7%) and the model recovered the correct hyper-parameters for the MNL model. The MNL model is nested in the economic screening rule model (via sufficiently low values of γ_h) and in this example the parameter estimates yielded the correct interpretation.

5. Empirical Application

In this section we describe data and results from a commercial marketing research study using the MNL, EBA, economic screening rule model, and a variant of the conjunctive screening rule model of Gilbride and Allenby (2004).

Data

The proposed models are illustrated using data from a discrete choice conjoint study. Due to the proprietary nature of the data, the actual products are disguised as are the specific product attributes and levels. The study's sponsor is a distributor of documentary films and was interested in consumers' preferences for specific titles and other features of documentaries that are to be sold for the home viewing market. The study involved 10 specific documentaries on different topics and it was thought that only a subset of the titles may appeal to any single consumer. Different prices, types of promotions, and other attributes were included in the study

to determine the trade-offs consumers would make. Altogether 7 attributes were included in the design: documentary title, media (DVD vs. VHS), packaging, two other disguised attributes, promotion, and price at four different levels. A complete list of attributes and levels is in table 3.

The data collection method included partial profile and full profile choice sets as well as recording choices in the "dual response format." Respondents evaluated a total of 16 choice sets – 10 partial profile choice sets which included six (of the 10) individual titles described on the remaining six attributes, and six full profile choice sets with each comprising the ten documentary titles, each listed in the DVD and VHS format, resulting in 20 alternatives to choose from. The remaining five attributes were also included in the full profile tasks. The complexity of the full profile task is similar to that encountered in the marketplace, and makes it likely that some of elimination based strategy or screening would take place.

Each respondent saw the same 16 choice sets: half evaluated the full profile and then the partial profile with the order reversed for the other half of the sample. The presentation order for all choice sets was randomized across respondents. For each choice set, the consumer was asked to indicate his/her preferred alternative. In a follow-up question, the consumer was asked if the selected alternative was available, would the consumer actually purchase it. This is an example of the "dual response format" introduced in commercial marketing research studies to mitigate respondents' proclivity to select the "none" option when it is included as one of the choice options. When "none" is selected with too great a frequency, the data becomes non-informative and parameter estimates are unstable.

The purpose of the empirical application is to illustrate the use of the proposed models with data commonly collected in marketing research studies. A variety of different models can be postulated as to how consumers move from indicating the most preferred option to a

(hypothetical) purchase decision. Instead of modeling this process, the data is analyzed once using the "forced choice" outcome as the dependent variable and a second time assuming "none" is an alternative in every choice set. In the second analysis, if the consumer indicated that he/she would not purchase the most preferred alternative, then "none" was determined to be the selected alternative. Otherwise, the most preferred alternative was used as the dependent variable. This dual analysis was done to illustrate the models using data collected with and without the "none" option as part of the marketing research study. Parameter estimates from the two analyses are discussed below.

Responses from 296 consumers are available for analysis. In the "forced choice" analysis, consumers made 10 choices from a set of six alternatives and 6 choices from a set of 20 alternatives. In the "none" analysis, respondents made 10 choices from a set of seven alternatives (6 titles + none) and 6 choices from a set of 21 (20 titles + none) alternatives. The "none" option was selected 13.8% of the time. In both treatments of the data, there are a total of $296 \times 16 = 4,736$ data points for model estimation.

Models

Four models are fit separately to data with the "forced choice" outcome and the "none" option. In addition to the EBA and economic screening rule models discussed above, a heterogeneous multinomial logit model and a conjunctive screening rule model are also estimated. The conjunctive screening rule model takes the following form:

$$\text{Choose alternative } i \text{ if } V_i > V_n \text{ for all } N \text{ such that } I(x_n, \tau) = 1 \quad (11a)$$

$$\text{where } I(x_n, \tau) = 1 \text{ if } \prod_j I(\tau_j > x_{nj}) = 1 \quad (11b)$$

Equation (11a) says to choose the alternative with the maximum indirect utility from those that pass the screening rule. (11b) says that for an alternative to pass the screening rule, it must not have any unacceptable attributes. For dummy coded variables, $x_{nj} \in \{0, 1\}$ then $\tau_j \in \{0.5, 1.5\}$. For $\tau_j = 0.5$, $I(0.5 > 0) = 1$, $I(0.5 > 1) = 0$ and therefore the presence of attribute x_{nj} is unacceptable. Alternatively, for $\tau_j = 1.5$, $I(1.5 > 0) = 1$, $I(1.5 > 1) = 1$ and x_{nj} is an acceptable attribute.

Similar to the model of Gilbride and Allenby (2004), the τ_j 's are augmented variables introduced to simplify the analysis. Conditional on the vectors of parameters β and τ , the conditional likelihood is identical to equation (10) for the economic screening rule model when the same assumptions regarding the error term are made. Heterogeneity is introduced for all parameters and for the augmented τ_{hj} via $\Pr(\tau_{hj} = 0.5) = \theta_j$ modeled across respondents. With binary variables, θ_j is interpreted as the probability that attribute j is unacceptable. As shown in Table 5, this results in 27 values of θ to estimate. Full details of the estimation algorithm are contained in the appendix.

Price is treated as a continuous variable in the indirect utility function [via $\ln(\text{Price}_i)$] but as an ordinal variable in the EBA and in the screening rules. In the EBA model, it is assumed that consumers select price based on a "less than or equal to" criteria: e.g. a consumer selects alternatives that are "less than or equal to Price2", any alternative with a price "greater than Price2" is eliminated. This treatment was suggested by Rotondo (1986). In the screening rule model, it is assumed that consumers screen prices based on a "greater than or equal to" criteria: e.g. alternatives "greater than or equal to Price2" are unacceptable, etc. For the conjunctive screening rule model, this implies the threshold for price is distributed multinomial across

respondents. For more information on coding variables for the conjunctive screening rule model see the explanation and examples in Gilbride and Allenby (2004).

The models were estimated using MCMC methods employing hybrid chains that used standard techniques. The EBA and MNL models were estimated using 20,000 iterations with a sample of every 10th iteration from the last 10,000 used to describe the posterior moments. The conjunctive screening rule model was estimated from a run of 50,000 iterations with a sample of every 10th from the final 10,000 used for inference. The economic screening rule model was run for 100,000 iterations again with a sample of every 10th from the final 10,000 used for inference. The economic screening rule model was run for a relatively large number of iterations because simulation studies showed that the parameter $\bar{\gamma}^*$ could be slow to converge; this was not a problem in the empirical applications, however. Convergence was assessed by starting the chains from multiple starting points and inspecting the time series plots of the parameters and identified functions of the parameters.

Results

Table 2 contains the fit statistics for the four models and the two different methods of handling the dual response format. As in the simulated data sets, the log marginal density (LMD) is computed using the importance sampling method of Newton and Raftery (1994). The economic screening rule model has the highest LMD for both the "forced choice" and "none" treatment of the data and is the favored model for this data. The fit of the conjunctive screening rule model is comparable, but slightly worse than the fit of the economic screening rule model. The EBA model did not fit the data as well as any of the other models although it did relatively better with

the "forced choice" than with the "none" option treatment of the data. Nonetheless, it is informative to look at the parameter estimates.

===== Table 2 =====

The EBA model provides an easy way to compare the relative importance of attributes and levels. Table 3 provides the posterior means (averaged across respondents and iterations of the MCMC chain) of the $\Pr(\{kj\}^l=1)$, that is the probability that attribute k, level j is selected as the first attribute/level used to eliminate alternatives. Focusing on the "none" option values, the posterior mean for title 1 being selected as the first attribute/level is .036 compared to .057 for title 2; this suggests title 2 is favored, on average, to title 1. Similarly, with a posterior mean of .146, being below Price2 is the attribute/level most likely to be chosen first to eliminate alternatives from the choice set. By summing $\sum_j (\Pr(\{kj\}^l=1))$ we can measure the relative importance of the attributes. The sum of the posterior means across the documentary titles is .644 compared to .156 for the different price points and .133 for the two types of media. The remaining attributes and attribute/levels have relatively low posterior means. As we move from the "forced choice" to the "none" treatment of the data, we find that the price points are relatively more important in the "forced choice" data and while the importance of "media" is about the same, the relative importance of VHS versus DVD is reversed.

===== Table 3 =====

Parameter estimates and functions of parameters for the economic screening rule model are presented in Table 4. With the exception of $\ln(\text{Price})$, dummy level coding was used to represent the values of the attributes/levels in the indirect utility function. In the "forced choice" treatment of the data, identification was accomplished by setting $\beta_{\text{Title}1}=0$. This resulted in 18 values of β_{hj} and one γ_h to be estimated for each individual. In data with a "none" option,

identification is achieved by setting the explanatory variables for the "none" option all equal to 0. This allows estimation of β_{Title1} and results in 19 values of β_{hj} and one γ_h to be estimated for each individual. In the "none" treatment of the data, all price points are greater than or equal to the lowest price, so it is not used as a screening attribute-level. The posterior means and posterior standard deviations of selected parameters from the distribution of heterogeneity are presented in the table. Note that since different identification schemes are used, the values for $\bar{\gamma}^*$, $\sigma_{\gamma^*}^2$, and $\bar{\beta}$ are not directly comparable across the data sets.

===== Table 4 =====

The columns labeled "% Unacceptable" are calculated by using the individual values of β_h and γ_h together with equation (9) to determine the acceptable and unacceptable levels of each attribute/level. The average is taken across individuals and across draws of the MCMC chain. These numbers are presented in % format to highlight the difference between them and estimates of θ in the conjunctive screening rule model. The "% Unacceptable" columns show that virtually all of the screening is done on the basis of the title of the documentary and the price. Also, as we move from the "forced choice" to the "none" treatment of the data, consumers are more likely to find any attribute/level unacceptable when "none" is part of the choice set. This makes sense, when the consumer is forced to choose, they are more likely to settle for an alternative that they otherwise might have screened out.

Table 5 contains summary statistics of parameters from the conjunctive screening rule model. The models are identified using the same methods as in the economic screening rule model and resulted in 18 and 19 individual level β_h parameters in the two data sets. In each dataset, there are 27 augmented values of τ_h resulting in an equivalent number of θ 's to describe

the distribution of heterogeneity. The θ 's are interpreted as the probability that the attribute/level is unacceptable across the sample.

===== Table 5 =====

The conjunctive screening rule model indicates that documentary title, media, and to a lesser extent, price are used to screen alternatives. The implied screening for media is similar for the economic and conjunctive screening rule models, but the economic screening rule model suggests much higher use of documentary titles and price to screen out alternatives. Comparing the "forced choice" to the "none" option treatment of the data, substantive changes in the conjunctive screening rule parameters are seen for Title 1 (0.259 vs. .097), Title 2 (.065 vs. .120), and Title 3 (.156 vs. .050) but the remaining parameters are comparable. This contrasts with the economic screening rule results where a general increase in the use of attribute/levels to screen out alternatives was observed when "none" was part of the choice set.

The empirical results support the conclusion that respondents were using a screening rule choice model as compared to an elimination based choice model. Further, the economic screening rule choice model is favored over the conjunctive screening rule model. The comparisons between the EBA and screening rule models are made based on similar treatments of heterogeneity, using the same datasets with and without a "none" option, and using specific attribute/levels of interest to marketing managers and product designers.

6. Discussion

This paper has presented methods for estimating two divergent theories of choice. The elimination by aspects theory postulates a sequential elimination of alternatives by attribute-levels until a single one, the chosen alternative remains. In the economic screening rule model,

respondents screen out alternatives with certain attribute levels, and then choose from the remaining alternatives using a compensatory function of all the attributes. The direct comparison of these, and other choice models, has been hampered by the difficulty of estimating them, with heterogeneity, for reasonably sized choice sets. As the number of alternatives and attribute levels increases, specifying the conditional choice probability in closed form as in equation (1) becomes more challenging. To deal with this, some researchers have described choice alternatives in abstract terms, limited the number of attributes, and/or estimated individual level models with non-representative samples. We demonstrate our methods using data from a commercial study with choice alternatives described on 26 specific attribute/levels, choice tasks comprising many choice alternatives, and the presence of a "none" choice alternative. Consistent with other modeling approaches, heterogeneity is modeled through a hierarchical Bayesian structure.

Attribute based screening rules have been proposed in the behavioral literature and statistical models estimated by Gilbride and Allenby (2004). In these models, consumers eliminate a subset of choice alternatives based on the levels of particular attributes, and then choose the preferred alternative from the reduced choice set. The economic screening rule model gives an economic justification as to why certain attributes are used to screen alternatives. The model calculates the expected loss in utility of screening out alternatives with a particular attribute level. Choice alternatives with that attribute level are screened out if the loss in expected utility is less than the cognitive cost of the evaluation process. This economic approach to modeling screening rules results in fewer parameters to estimate as compared to a comparable conjunctive screening rule model. Our empirical study indicates support for this model relative to the standard MNL model, the EBA model and a model with a conjunctive screening rule.

Several extensions of this study are possible. The empirical application in this study represents a single data point and additional research with many data sets is needed to understand when and under what conditions consumers may adopt different decision strategies. The implications of using the dual response format are not well known and it may predispose respondents to make compensatory trade-offs via the framing of the follow-up question "would you actually buy this alternative." If true, this would bias against the EBA model and favor the standard economic or multinomial logit model. Given the large number of individual level parameters and the disparity between different models (26 individual level parameters in the EBA model, 19 in the economic screening rule, and 45 in the conjunctive screening rule), all the available data was used to estimate the models leaving none for hold-out sample validation. Additional data sets with suitably designed hold-out samples will provide model validation beyond just the in-sample fit. The methods and models in this paper are appropriate for investigating these issues.

Two theoretical extensions of the economic screening rule model are also possible. First, the model conditions on a consumer's current beliefs about the distribution of attributes and indirect utility across choice alternatives. The model could be expanded by including a dynamic component where consumers update their beliefs, see for instance Erdem and Keane (1996). Secondly, in the empirical application, the "none" option was treated as another alternative with null values for the attributes. Additional insight may be provided by explicitly modeling the budget constraint and the choice of the "outside good."

It is hoped that the models and methods proposed in this paper will be used by researchers to investigate choice processes in settings typically encountered in marketing problems. Additional empirical research is needed to understand if consumers adopt a variety of

choice models and under what conditions, or if a single parsimonious representation is sufficient to characterize consumer choice.

Appendix A Estimation of EBA Choice Probabilities

Calculating the choice probability in (4) is facilitated by using the Modified Latin Hypercube Sampling (MLHS) plan as proposed by Hess, Train, and Polak (2005). Using quasi-random number sequences such as the MLHS can provide comparable results to random number sequences but with many fewer simulations. The order in which attribute levels $\{k_j\}^t$ are selected in step i) of the algorithm is governed by the draw of uniform (0,1) random variables across the K attributes and S simulations. The S simulations in equation (4) approximate:

$$\int_0^1 g(u) f(u) du \tag{A.1}$$

where $g(u)$ is the selection algorithm given by i) to iii) and $f(u)$ is a multivariate uniform density of dimension K . One way to estimate (4) is to generate $S \times K$ independent uniform random numbers. This produces an unbiased estimator of $\Pr(y_{im}=1)$ whose variance and simulation error decreases as S increases.

The advantage to quasi-random number sequences is that they improve the coverage over the area of integration, e.g. the unit interval. Bearing in mind that computer packages generate only pseudo-random numbers, for a finite number of simulations S , it is possible to have the draws "clustered" in the low end of the unit interval, or at the high end of the unit interval, etc. As S increases, the uniform distribution is better approximated and the coverage is improved. The MLHS is designed to ensure better coverage for any finite number of simulations by

breaking up the "unit hyper-cube" into evenly spaced intervals. A one dimensional sequence of length S is created by:

$$\varphi(s) = \frac{s-1}{S}, s = 1, \dots, S \quad (\text{A.2})$$

The sequence is then randomly re-ordered (shuffled). A multidimensional array of size K is constructed by combining the one-dimensional arrays. The final step is to add a pseudo-random number $0 < u < 1/S$ to each element:

$$\vartheta_k(s) = \varphi_k(s) + u_k \quad s = 1, \dots, S, \quad k=1, \dots, K \quad (\text{A.3})$$

See Hess et al. (2005) for more details and a comparison of the MLHS to other quasi-random plans such as the Halton sequence.

Simulations were conducted to measure the ability of the MLHS plan to estimate EBA choice probabilities. Attribute level weights were assigned and EBA choices were generated for 10 individuals across 20 choice occasions. Each choice set contained 10 alternatives and each alternative was described on 15 binary attributes. Pseudo-Random draws were used to generate the actual choices. Using the actual attribute level weights, α_{hkj} , the algorithm i) to iii), and equation (4) were used to determine how well the "true" choice probabilities could be recovered using various sampling plans. The "true" choice probabilities were calculated using pseudo-random numbers and $S = 5,000$ for each of the (10 individuals \times 20 choice occasions) = 200

probabilities. This process was repeated 100 times and the average was used as the "true" choice probability.

Simulations using pseudo-random numbers and the MLHS plan with $S = 100, 250, 500$ and 1,000 were then conducted. For example, the choice probability for each of the 200 probabilities was calculated using the MLHS plan with $S = 100$ and the squared difference between the "true" and the predicted probability was recorded. This was replicated 100 times and the average was calculated across replications and choice probabilities to obtain the mean square error. The square root of the mean square errors are presented in Figure 1. As expected, the root mean square error decreases as S increases for both the pseudo-random number and MLHS plans. It is clear, though, that the MLHS has a uniformly lower root mean square error. Based on these results, $S = 500$ and the MLHS plan were used in the empirical applications.

===== Figure 1 =====

Appendix B Estimation Algorithms

This appendix provides detailed algorithms for estimating the models in the paper. Except where noted, the notation follows that introduced in the text.

EBA Model

The model hierarchy is given by:

$$y|\alpha, X \quad (B1)$$

$$\alpha^* | \overline{\alpha^*}, V_{\alpha^*} \quad (B2)$$

$$\overline{\alpha^*} \quad (B3)$$

$$V_{\alpha^*} \quad (B4)$$

(B1) is the likelihood given by i) to iii) and equation (4) in the paper where $\alpha = \exp(\alpha^*)$, (B2) describes the distribution of heterogeneity $\alpha_h^* \sim N(\overline{\alpha^*}, V_{\alpha^*})$, and (B3), and (B4) are conjugate but diffuse priors on the hyper-parameters:

$$\overline{\alpha^*} \sim N(0, 100I)$$

$$V_{\alpha^*} \sim IW(\nu, \Delta)$$

Where I represents the identity matrix, IW is the Inverted Wishart distribution with $\nu = V + 8$, $V=26$ parameters, and $\Delta = \nu I$. The following steps describe a MCMC chain with the posterior distribution of all model parameters as the stationary distribution.

1. Generate $\alpha_h^* | \overline{\alpha^*}, V_{\alpha^*}, y_h, X$ for $h=1 \dots H$

A random-walk Metropolis-Hastings step is used. Let $\alpha_h^{*(p)}$ represent the proposed candidate new draw and $\alpha_h^{*(o)}$ represent the current or old draw. Form $\alpha_h^{*(p)} = \alpha_h^{*(o)} + \eta$ where η is a draw from $N(0, zV_{\alpha^*})$ and z is a scalar multiple selected to ensure an approximate 50% rejection rate. Form $\alpha_h^{(p)} = \exp(\alpha_h^{*(p)})$.

For $m=1, \dots, M$ (choice sets, $M=16$ in this application):

a) Generate the MLHS quasi-random numbers (described in Appendix A):

$$\vartheta_k(s) = \varphi_k(s) + u_k \quad s = 1, \dots, S, \quad k=1, \dots, K$$

Where $S = 500$ total simulations, and $K = 7$ attributes

b) Calculate the simulated choice probability:

For $s = 1, \dots, S$ and $t = 1$ to K .

- iv) Using $\vartheta_k(s)$, select attribute level $\{kj\}^t$ with probability $\frac{\alpha_{hkj}^{(p)}}{\sum_{k=1}^K \sum_{j=1}^{J_k} \alpha_{hkj}^{(p)}}$
- v) Eliminate all alternatives that do not have attribute level $\{kj\}^t$
- vi) If only one alternative, i , remains set $y_{hims}^* = 1$; otherwise increase t and go to i)

where the notation $\{k^{t-1}, k^{t-2}, \dots\}$ represents the set of previously selected attributes.

$$l_{hm}^{(p)} = \Pr(y_{him}=1) = \frac{\sum_{s=1}^S I(y_{hims} = y_{hims}^*)}{S}$$

c) Accept or reject the proposed vector $\alpha_h^{*(p)}$ with probability:

$$\text{Min:} \left(\frac{\prod_{m=1}^M l_{hm}^{(p)} \times \exp\left(-\frac{1}{2}(\alpha_h^{(p)*} - \bar{\alpha}^*)V_{\alpha^*}^{-1}(\alpha_h^{(p)*} - \bar{\alpha}^*)\right)}{\prod_{m=1}^M l_{hm}^{(o)} \times \exp\left(-\frac{1}{2}(\alpha_h^{(o)*} - \bar{\alpha}^*)V_{\alpha^*}^{-1}(\alpha_h^{(o)*} - \bar{\alpha}^*)\right)}, 1 \right)$$

d) Post-process the accepted draws:

For $j=1, \dots, J_k$ and $k=1, \dots, K$

$$\Pr_h(\{kj\}^1) = \frac{\alpha_{hkj}}{\sum_{k=1}^K \sum_{j=1}^{J_k} \alpha_{hkj}}$$

2. Generate $\bar{\alpha}^* | \{\alpha_h^*\}, V_{\alpha^*}$

$$\bar{\alpha}^* \sim N(\bar{a}, ((V_{\alpha^*}/H)^{-1} + (100I)^{-1})^{-1})$$

$$\bar{\mathbf{a}} = ((V_{\alpha^*}/H)^{-1} + (100\mathbf{I})^{-1})^{-1} (V_{\alpha^*}^{-1} \sum_{h=1}^H \alpha_h^* + (100\mathbf{I})^{-1}(\mathbf{0}))$$

3. Generate $V_{\alpha^*} \mid \{\alpha_h^*\}, \bar{\alpha}^*$

$$V_{\alpha^*} \sim \text{IW}(v + H, \Delta + \sum_{h=1}^H (a_h^* - \bar{\alpha}^*)(\alpha_h^* - \bar{\alpha}^*))$$

Economic Screening Rule Model

The model hierarchy is given by:

$$y \mid \beta, \gamma, X \quad (\text{B5})$$

$$\beta \mid \bar{\beta}, V_{\beta} \quad (\text{B6})$$

$$\gamma^* \mid \bar{\gamma}^*, \sigma_{\gamma^*}^2 \quad (\text{B7})$$

$$\bar{\beta} \quad (\text{B8})$$

$$V_{\beta} \quad (\text{B9})$$

$$\bar{\gamma}^* \quad (\text{B10})$$

$$\sigma_{\gamma^*}^2 \quad (\text{B11})$$

(B5) is the likelihood function given in the text by equation (10), (B6) and (B7) are Normal distributions for heterogeneity, and (B8) to (B11) are conjugate but diffuse priors on the hyper-parameters:

$$\bar{\beta} \sim \text{N}(0, 100\mathbf{I}) \quad \bar{\gamma}^* \sim \text{N}(0, 100)$$

$$V_{\beta} \sim \text{IW}(v, \Delta) \quad \sigma_{\gamma^*}^2 \sim \text{IG}(a, b)$$

Where the IG represent the inverse gamma distribution, a=10 and b=.1 and the other notation is as above. The following steps describe a MCMC chain with the posterior distribution of all model parameters as the stationary distribution.

1. Generate $\beta_h \mid \gamma_h, \bar{\beta}, V_\beta, y_h, X$ for $h=1, \dots, H$

A random-walk Metropolis-Hastings step is used. Let $\beta_h^{(p)}$ represent the proposed candidate new draw and $\beta_h^{(o)}$ represent the current or old draw. Form $\beta_h^{(p)} = \beta_h^{(o)} + \eta$ where η is a draw from $N(0, zV_\beta)$ and z is a scalar multiple selected to ensure an approximate 50% rejection rate. Let j indicate the dummy coded variables used in the indirect utility function: $J=18$ for "forced choice" and $J=19$ for "none" option data; let j' indicate the specific attribute levels: $J' = 26$ for "forced choice" and $J' = 27$ for "none" option data [see text and table 4].

a) Determine which attributes are unacceptable:

For $j' = 1, \dots, J'$

$d_{j'} = 1$ if

$$\frac{1}{M} \sum_{m=1}^M \left\{ \ln \sum_{n=1}^{N_m} \exp \left(\sum_{j=1}^J \beta_{hj}^{(p)} x_{nj} + \beta_{hp}^{(p)} \ln(p_n) \right) - \ln \sum_{n^*=1}^{N_m^*} \exp \left(\sum_{j=1}^J \beta_{hj}^{(p)} x_{n^*j} + \beta_{hp}^{(p)} \ln(p_{n^*}) \right) \right\} < \gamma_h$$

Otherwise $d_{j'} = 0$. n^* indicates alternatives without attribute level j' .

b) Determine which alternatives pass the screening rule and calculate the likelihood:

For $m=1, \dots, M$ (*choice sets*)

For $n=1, \dots, N_m$ (*alternatives in choice sets*)

$I_n = 1$ if

$$\sum_{j'=1}^{J'} (d_{j'} \times x_{nj'}) = 0, \text{ otherwise } I_n = 0$$

If $y_{hnm} = 1$ and $I_n = 0$, then reject $\beta_h^{(p)}$, else:

$$l_{hm}^{(p)} = \Pr(y_{hnm}=1) = \frac{\exp \left(\sum_{j=1}^J \beta_{hj}^{(p)} x_{ij} + \beta_{hp}^{(p)} \ln(p_i) \right)}{\sum_{\substack{n=1 \\ I_n=1}}^{N_m} \exp \left(\sum_{j=1}^J \beta_{hj}^{(p)} x_{nj} + \beta_{hp}^{(p)} \ln(p_n) \right)}$$

c) If the proposed vector $\beta_h^{(p)}$ was not rejected in step b), then accept $\beta_h^{(p)}$ with probability:

$$\text{Min:} \left(\frac{\prod_{m=1}^M l_{hm}^{(p)} \times \exp\left(-\frac{1}{2}(\beta_h^{(p)} - \bar{\beta})V_{\beta}^{-1}(\beta_h^{(p)} - \bar{\beta})\right)}{\prod_{m=1}^M l_{hm}^{(o)} \times \exp\left(-\frac{1}{2}(\beta_h^{(o)} - \bar{\beta})V_{\beta}^{-1}(\beta_h^{(o)} - \bar{\beta})\right)}, 1 \right)$$

2. Generate $\gamma_h^* | \beta_h, \bar{\gamma}^*, \sigma_{\gamma^*}^2, y_h, X$ for $h=1, \dots, H$

A random-walk Metropolis-Hastings step is used. Let $\gamma_h^{*(p)}$ represent the proposed candidate new draw and $\gamma_h^{*(o)}$ represent the current or old draw. Form $\gamma_h^{*(p)} = \gamma_h^{*(o)} + \eta$ where η is a draw from $N(0, z1)$ and z is a scalar multiple selected to ensure an approximate 50% rejection rate.

Form $\gamma_h^{(p)} = \exp(\gamma_h^{*(p)})$.

a) & b) Follow the steps for drawing β_h using the current β_h and replacing γ_h with $\gamma_h^{(p)}$. $\gamma_h^{(p)}$ is rejected in step b) if it results in a selected alternative being screened out of the choice set.

c) If the proposed scalar $\gamma_h^{(p)}$ was not rejected in step b), then accept $\gamma_h^{(p)}$ with probability:

$$\text{Min:} \left(\frac{\prod_{m=1}^M l_{hm}^{(p)} \times \exp\left(-(2\sigma_{\gamma^*}^2)^{-1}(\gamma_h^{*(p)} - \bar{\gamma}^*)\right)}{\prod_{m=1}^M l_{hm}^{(o)} \times \exp\left(-(2\sigma_{\gamma^*}^2)^{-1}(\gamma_h^{*(o)} - \bar{\gamma}^*)\right)}, 1 \right)$$

Draws of the hyper-parameters follow standard conjugate set-ups and are not detailed here.

Conjunctive Screening Rule

The model hierarchy is given by:

$$y | \beta, \tau, X \quad (B12)$$

$$\beta | \bar{\beta}, V_{\beta} \quad (B13)$$

$$\tau | \theta \quad (B14)$$

$$\bar{\beta} \quad (B15)$$

$$V_{\beta} \quad (B16)$$

$$\theta \quad (B17)$$

(B12) is the likelihood function and (B13) describes a Normal distribution of heterogeneity. τ_{hj} are distributed Bernoulli with $\Pr(\tau_{hj} = 0.5) = \theta_j$. For the price variable, $\tau_{h,price}$ can take on 4 discrete values and $\tau_{h,price} \sim \text{Multinomial}(H, \theta_{\text{Allacceptable}}, \theta_{\geq \text{Price}2}, \theta_{\geq \text{Price}3}, \theta_{\geq \text{Price}4})$. Prior distributions on the hyper-parameters are given by:

$$\begin{aligned} \bar{\beta} &\sim N(0, 100I) & V_{\beta} &\sim IW(\nu, \Delta) \\ \theta_j &\sim \text{Beta}(d, e) & \theta_{\text{price}} &\sim \text{Dirichlet}(\delta) \end{aligned}$$

Where $d = e = 3$ and δ is vector of length 4 with each element set = 4. The following steps describe a MCMC chain with the posterior distribution of all model parameters as the stationary distribution.

1. *Generate* $\tau_h | \theta, \beta_h, y_h, X$ for $h=1, \dots, H$

Similar to the algorithm described by Gilbride and Allenby (2004), a "Griddy Gibbs" step is used. Let j indicate the dummy coded variables used in the indirect utility function: $J=18$ for "forced choice" and $J=19$ for "none" option data; let j' indicate the specific attribute levels: $J' = 27$ for both data sets [see text and table 5].

For $j' = 1, \dots, J'=23$ (*exclude price*)

$\tau_{hj'}$ can be equal to one of two values: $\tau_{hj'}^{(0.5)} = 0.5$ and $\tau_{hj'}^{(1.5)} = 1.5$. Conditioning on all other τ_h :

- a) Determine which alternatives pass the screening rule and calculate the likelihood for $\tau_{hj'}^{(0.5)}$.

For $m=1, \dots, M$ and $n=1, \dots, N_m$

Let $I_n = 1$ if:

$$\prod_{j'} I(\tau_{hj'}^{(0.5)} > x_{nj'}) = 1, \text{ otherwise } I_n = 0$$

If $y_{hnm} = 1$ and $I_n = 0$ reject $\tau_{hj'}^{(0.5)}$, else

$$l_{hm}^{(0.5)} = \Pr(y_{hnm}=1) = \frac{\exp\left(\sum_{j=1}^J \beta_{hj} x_{ij} + \beta_{hp} \ln(p_i)\right)}{\sum_{\substack{n=1 \\ I_n=1}}^{N_m} \exp\left(\sum_{j=1}^J \beta_{hj} x_{nj} + \beta_{hp} \ln(p_n)\right)}$$

$$l_h^{(0.5)} = \prod_{m=1}^M l_{hm}^{(0.5)}$$

- b) Determine which alternatives pass the screening rule and calculate the likelihood for $\tau_{hj'}^{(1.5)}$. This step follows the same procedure as a), but note that $\tau_{hj'}^{(1.5)}$ will never be rejected as inconsistent with the observed choices. Recall that $\tau_{hj'} = 1.5$ means that both the presence and absence of an attribute is acceptable.
- c) Select $\tau_{hj'}^{(0.5)}$ with probability:

$$\frac{l_h^{(0.5)} \times \theta_{j'}}{(l_h^{(0.5)} \times \theta_{j'}) + (l_h^{(1.5)} \times (1 - \theta_{j'}))} \text{ otherwise set } \tau_{hj'} = 1.5.$$

For $j' = 24, \dots, J'$ (the price variable)

The multinomial outcomes are selected using a procedure analogous to the one outlined above.

2. Generate $\beta_h \mid \tau_h, \bar{\beta}, V_\beta, y_h, X$ for $h=1, \dots, H$

A random-walk Metropolis-Hastings step is used. Let $\beta_h^{(p)}$ represent the proposed candidate new draw and $\beta_h^{(o)}$ represent the current or old draw. Form $\beta_h^{(p)} = \beta_h^{(o)} + \eta$ where η is a draw from $N(0, zV_\beta)$ and z is a scalar multiple selected to ensure an approximate 50% rejection rate.

For $m=1, \dots, M$ and $n=1, \dots, N_m$

Let $I_n = 1$ if:

$$\prod_j I(\tau_{hj} > x_{nj}) = 1, \text{ otherwise } I_n = 0$$

$$l_{hm}^{(p)} = \Pr(y_{him}=1) = \frac{\exp\left(\sum_{j=1}^J \beta_{hj}^{(p)} x_{ij} + \beta_{hp}^{(p)} \ln(p_i)\right)}{\sum_{\substack{n=1 \\ I_n=1}}^{N_m} \exp\left(\sum_{j=1}^J \beta_{hj}^{(p)} x_{nj} + \beta_{hp}^{(p)} \ln(p_n)\right)}$$

Accept $\beta_h^{(p)}$ with probability:

$$\text{Min:} \left(\frac{\prod_{m=1}^M l_{hm}^{(p)} \times \exp\left(-\frac{1}{2}(\beta_h^{(p)} - \bar{\beta})V_\beta^{-1}(\beta_h^{(p)} - \bar{\beta})\right)}{\prod_{m=1}^M l_{hm}^{(o)} \times \exp\left(-\frac{1}{2}(\beta_h^{(o)} - \bar{\beta})V_\beta^{-1}(\beta_h^{(o)} - \bar{\beta})\right)}, 1 \right)$$

Draws of the hyper-parameters follow standard conjugate set-ups and are not detailed here. See especially Gilbride and Allenby (2004).

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Figure 1
Recovery of EBA Choice Probabilities
MLHS compared to Pseudo-Random Number Generation

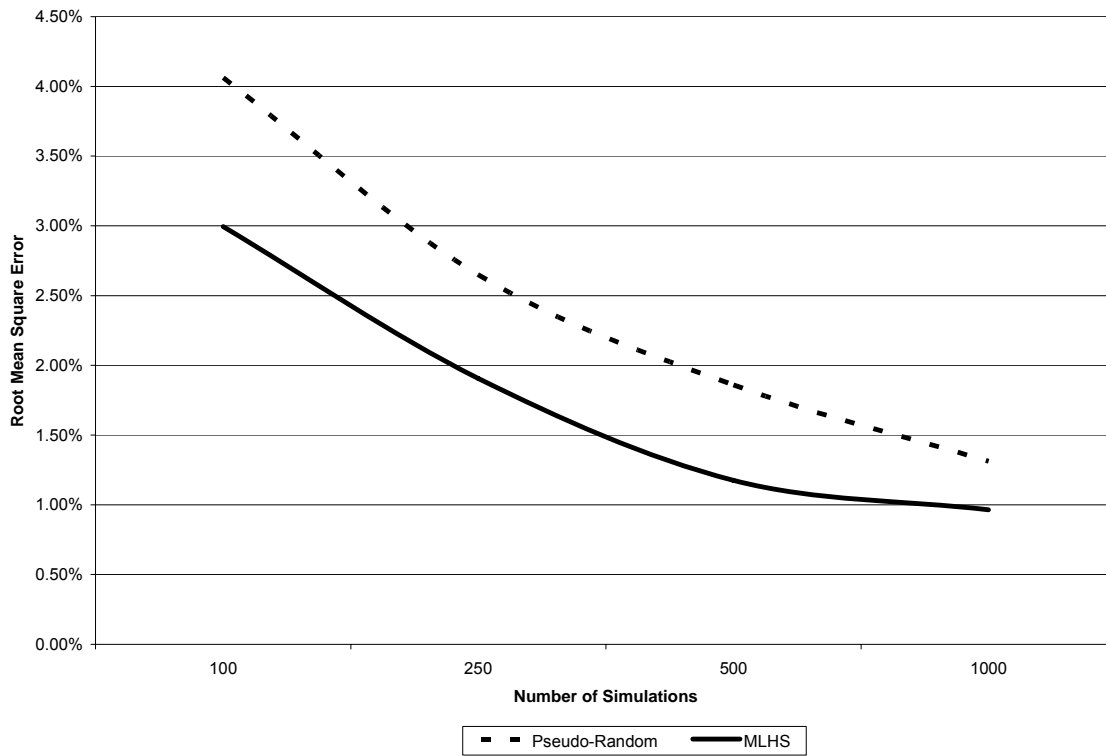


Table 1
Log Marginal Density using Simulated Data¹

		Model used to estimate parameters		
		EBA	MNL	Economic Screening Rule
Data generating process	EBA	-4,150.30	-4,219.40	-4,235.40
	MNL	-3,048.40	-2,964.50	-2,957.50
	Economic Screening Rule	-2,975.90	-2,963.10	-2,819.40

¹The log marginal density calculated using the importance-sampling method of Newton and Raftery (1994) is in each cell. The data favors the model with the largest log marginal density.

Table 2
Model Results: LMD¹

	<u>Forced Choice</u>	<u>"None" option</u>
EBA	-4,912.0	-6,919.2
MNL	-4,331.7	-4,622.7
Economic Screening Rule	-4,176.3	-4,470.0
Conjunctive Screening Rule	-4,207.6	-4,526.4

¹The log marginal density calculated using the importance-sampling method of Newton and Raftery (1994) is in each cell.

Table 3
Elimination by Aspects Model
Function of parameters: $\Pr(\{kj\}^1)$

	Forced Choice		"None" Option	
	Posterior Mean	Posterior Std. Dev.	Posterior Mean	Posterior Std. Dev.
Documentary				
Title 1	0.031	(0.003)	0.036	(0.002)
Title 2	0.046	(0.003)	0.057	(0.004)
Title 3	0.027	(0.002)	0.034	(0.003)
Title 4	0.078	(0.004)	0.083	(0.004)
Title 5	0.104	(0.004)	0.099	(0.004)
Title 6	0.048	(0.003)	0.051	(0.003)
Title 7	0.065	(0.004)	0.079	(0.004)
Title 8	0.035	(0.003)	0.036	(0.002)
Title 9	0.043	(0.003)	0.057	(0.003)
Title 10	0.113	(0.005)	0.111	(0.005)
Media				
DVD	0.072	(0.005)	0.055	(0.006)
VHS	0.040	(0.009)	0.078	(0.010)
Packaging				
Packaging1	0.010	(0.002)	0.013	(0.003)
Packaging2	0.001	(0.000)	0.001	(0.001)
Attribute 4				
Option1	0.002	(0.001)	0.006	(0.002)
Option2	0.017	(0.003)	0.015	(0.003)
Attribute 5				
Option1	0.000	(0.000)	0.000	(0.000)
Option2	0.013	(0.003)	0.017	(0.002)
Promotion				
None	0.001	(0.000)	0.001	(0.000)
Promo1	0.007	(0.001)	0.007	(0.001)
Promo2	0.001	(0.000)	0.002	(0.001)
Promo3	0.001	(0.001)	0.003	(0.001)
Promo4	0.002	(0.001)	0.003	(0.001)
Price				
≤ Price1	0.080	(0.007)	0.005	(0.004)
≤ Price2	0.093	(0.007)	0.146	(0.011)
≤ Price3	0.070	(0.012)	0.005	(0.002)
≤ Price4				

Table 4
Economic Screening Rule Model
Selected Parameter Estimates and Functions of Parameters¹

	Forced Choice				"None" Option			
	$\bar{\beta}$		% Un-acceptable	(0.00)	$\bar{\beta}$		% Un-acceptable	(0.00)
Documentary								
Title 1	0	(0.00)	43.6%	(0.04)	3.927	(0.38)	51.0%	(0.03)
Title 2	1.236	(0.29)	25.0%	(0.03)	4.985	(0.36)	32.8%	(0.03)
Title 3	0.615	(0.30)	33.4%	(0.04)	4.527	(0.35)	39.0%	(0.04)
Title 4	0.908	(0.29)	33.6%	(0.03)	4.700	(0.35)	38.0%	(0.03)
Title 5	1.220	(0.36)	35.1%	(0.03)	5.001	(0.40)	39.6%	(0.03)
Title 6	-0.401	(0.33)	44.9%	(0.03)	3.511	(0.40)	51.5%	(0.03)
Title 7	2.100	(0.25)	15.7%	(0.03)	5.716	(0.37)	23.4%	(0.02)
Title 8	-0.429	(0.38)	55.0%	(0.04)	3.259	(0.45)	61.5%	(0.03)
Title 9	0.696	(0.30)	31.4%	(0.03)	4.572	(0.37)	38.4%	(0.04)
Title 10	2.330	(0.28)	17.5%	(0.03)	5.905	(0.39)	23.9%	(0.02)
Media								
DVD	-0.139	(0.14)	1.8%	(0.01)	-0.143	(0.14)	5.0%	(0.01)
VHS			1.0%	(0.01)			1.9%	(0.01)
Packaging								
Packaging1	0.305	(0.06)	0.0%	(0.00)	0.273	(0.07)	0.0%	(0.00)
Packaging2			0.0%	(0.00)			0.1%	(0.00)
Attribute 4								
Option1	-0.092	(0.07)	0.0%	(0.00)	-0.069	(0.07)	0.1%	(0.00)
Option2			0.0%	(0.00)			0.0%	(0.00)
Attribute 5								
Option1	-0.385	(0.07)	0.0%	(0.00)	-0.415	(0.07)	0.2%	(0.00)
Option2			0.0%	(0.00)			0.0%	(0.00)
Promotion								
None			0.3%	(0.00)			1.7%	(0.01)
Promo1	0.458	(0.10)	0.0%	(0.00)	0.537	(0.10)	0.4%	(0.00)
Promo2	-0.052	(0.10)	0.2%	(0.00)	-0.025	(0.10)	1.8%	(0.01)
Promo3	0.136	(0.10)	0.1%	(0.00)	0.185	(0.10)	1.2%	(0.00)
Promo4	0.304	(0.10)	0.1%	(0.00)	0.304	(0.11)	1.3%	(0.01)
ln(Price)								
≥ Price1	-2.483	(0.15)			-2.742	(0.16)	0.0%	(0.00)
≥ Price2			0.0%	(0.00)			0.9%	(0.00)
≥ Price3			0.6%	(0.00)			4.2%	(0.01)
≥ Price4			11.4%	(0.02)			22.2%	(0.02)
Cognitive cost parameters								
$\bar{\gamma}$	-4.102	(0.29)			-3.920	(0.23)		
σ_{γ}^2	0.613	(0.25)			0.956	(0.30)		

¹"Unacceptable" is a function of the parameters β and γ .
Posterior Means (Posterior Standard Deviations) are presented.

Table 5
Conjunctive Screening Rule Model
Selected Parameter Estimates¹

	Forced Choice				"None" Option			
	$\bar{\beta}$		θ		$\bar{\beta}$		θ	
Documentary								
Title 1	0	(0.00)	0.259	(0.06)	3.431	(0.43)	0.097	(0.06)
Title 2	0.503	(0.41)	0.065	(0.04)	5.062	(0.34)	0.120	(0.04)
Title 3	0.136	(0.43)	0.156	(0.07)	4.061	(0.34)	0.050	(0.04)
Title 4	0.374	(0.42)	0.105	(0.06)	4.669	(0.36)	0.082	(0.05)
Title 5	0.463	(0.54)	0.051	(0.04)	4.677	(0.44)	0.032	(0.03)
Title 6	-1.400	(0.63)	0.100	(0.06)	3.233	(0.43)	0.098	(0.05)
Title 7	1.438	(0.33)	0.035	(0.03)	5.614	(0.36)	0.036	(0.03)
Title 8	0.292	(0.60)	0.388	(0.08)	4.893	(0.51)	0.433	(0.06)
Title 9	0.360	(0.34)	0.163	(0.05)	4.978	(0.40)	0.196	(0.05)
Title 10	2.037	(0.40)	0.096	(0.03)	5.855	(0.42)	0.026	(0.02)
Media								
DVD	-0.018	(0.14)	0.041	(0.01)	-0.081	(0.14)	0.045	(0.02)
VHS			0.019	(0.01)			0.035	(0.01)
Packaging								
Packaging1	0.333	(0.07)	0.003	(0.00)	0.317	(0.07)	0.003	(0.00)
Packaging2			0.003	(0.00)			0.006	(0.00)
Attribute 4								
Option1	-0.054	(0.08)	0.010	(0.01)	-0.051	(0.08)	0.005	(0.00)
Option2			0.003	(0.00)			0.004	(0.00)
Attribute 5								
Option1	-0.398	(0.07)	0.006	(0.00)	-0.424	(0.07)	0.006	(0.01)
Option2			0.003	(0.00)			0.003	(0.00)
Promotion								
None			0.007	(0.01)			0.007	(0.01)
Promo1	0.550	(0.10)	0.004	(0.00)	0.703	(0.10)	0.004	(0.00)
Promo2	-0.042	(0.10)	0.005	(0.01)	0.037	(0.09)	0.006	(0.01)
Promo3	0.209	(0.11)	0.004	(0.00)	0.324	(0.11)	0.005	(0.00)
Promo4	0.376	(0.09)	0.006	(0.01)	0.474	(0.09)	0.008	(0.01)
ln(Price)								
≥ Price2	-2.518	(0.16)	0.029	(0.01)	-2.788	(0.16)	0.032	(0.01)
≥ Price3			0.026	(0.01)			0.023	(0.01)
≥ Price4			0.024	(0.01)			0.031	(0.01)
All prices acceptable			0.921	(0.02)			0.914	(0.02)

¹Except where noted for price, θ is the probability that the attribute/level is unacceptable. Posterior Means (Posterior Standard Deviations) are presented.